I've passed my quals, now what?

A guide for Ph.D. candidates in Mathematics

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Introduction

What is this?

Mathematics is a profession where for the most part we are on our own to figure out how to function. We are taught plenty of mathematics, but other than that we are left to our own devices. No one tells us how to find problems to work on or how to formulate conjectures, no one tells us how to write a paper or get it published, and certainly no one tells us how to conduct research.

This guide was written in an effort to provide suggestions for students as they become involved in the process of doing research and writing a thesis. After passing qualifying exams, it is not uncommon for students to feel as if they are set adrift, to figure out on their own how to get started on research and to produce a thesis by the end of the remaining years in graduate school. It has been my experience that this is a time when many people drop out of graduate school. Conducting original research is a process that is vastly different from taking classes and studying for qualifying exams. If a timely transition is not made, a student is at high risk for leaving the program without a Ph.D. The goal of this guide is to provide suggestions on avoiding pitfalls and maximizing productivity during the period of graduate school after passing qualifying exams and before graduation.

There are several excellent sources that provide advice on surviving graduate school and writing theses. Many of these can be found on the web and I suggest you look at them. The point of this particular guide is threefold: First, I want to collect in one place those suggestions that I found most useful when I was a graduate student. Second, since many of the existing guides are for other disciplines, I want to talk specifically about research in mathematics. Third, I want add some advice from my own experiences in mathematics departments at Dartmouth College, the University of Iowa, and the University of Houston.

Disclaimer: Throughout my mathematical career I have gotten advice from countless others. Consequently, much of what is found here is not original and very few of the ideas are mine. I am merely collecting suggestions and advice that I have heard from others and read in various sources. In some cases I have even taken information off of the internet and presented it here with few changes.

It goes without saying that my choice of what to include was influenced by my experiences and personal philosophies. What is in these notes are suggestions I have collected because I have found them useful and believe others may also. I am sure that other people will find parts that are not useful for them. That is fine. There are as many ways to do mathematics as there are mathematicians, so if you disagree with anything here feel free to ignore it. Or better yet, write down your own suggestions and either add them to this guide or create your own.

How to use this guide

There is no guaranteed recipe for success at research. This document consists of a rough description of the stages involved in the last few years of the mathematics Ph.D. program and collects some informal advice that may help during this time. The goal is to give the reader an idea of what to expect so that they can start preparing for events long before they come. It is probably best to not read this in one sitting, but rather to browse. One may also find it useful to skim the table of contents and then to refer to those sections relevant to one's current situation.

The first six chapters are organized into a rough chronology of stages that one goes through in writing a dissertation.

Chapter 1 is about choosing an advisor and research area.
Chapter 2 is about getting grounded in one's research area through reading.
Chapter 3 is about specializing and beginning a research project.
Chapter 4 is about doing research.
Chapter 5 is about becoming part of the research community.
Chapter 6 is about writing up results, including how to write a thesis.
Chapter 7 is about emotional factors in the process of research.

The guide concludes with two appendices containing miscellaneous suggestions.

Appendix A contains a list of myths about mathematics and graduate school.
Appendix B contains writing tips on mechanics, style, and use of TeX.

Many thanks to Susan D'Agostino for reading preliminary drafts of Version 1 of this guide and providing numerous suggestions. It has been improved considerably due to her efforts.

Versions

• Version 1 of this guide was written in Spring of 2002 while the author was a graduate student at Dartmouth College.

• Version 2 of this guide was written in Spring of 2003 while the author was a postdoc at the University of Iowa.

• Version 3 of this guide was written in Fall of 2007 while the author was an assistant professor at the University of Houston.

Chapter 1

Choosing an advisor

The first step after passing your quals is to choose an area and an advisor in that area with whom to work. This is a very important decision that you must make. Your advisor is your advisor for life. It is likely that you will stay in contact with this person even after you graduate and get a job, so the choice you make can have lasting effects. It is therefore unfortunate that one must make this decision at a time when they are often lacking the knowledge to choose well.

There is great danger in choosing a bad advisor. I have seen many good students who have dropped out in their last years due to a bad choice of advisor, and I have seen even more who have switched advisors and consequently lost years of work. Therefore, you should take the decision of your advisor very seriously.

So how should you go about choosing an advisor? Some obvious considerations are that you should get along with this person and that they should work in an area you are interested in. However, there are also other subtle considerations. Below you will find some of the factors that students consider (or wish they considered) when choosing their advisor. I have listed them in two categories. Primary factors identify qualities of an advisor that are essential for success, while secondary factors are qualities that are beneficial but not essential.

Primary Factors in Choosing an Advisor

- Has research interests in common with you.

  You of course want to work in an area you find interesting and fun. However, realize that this is just one of many equally important factors to consider. If you came to graduate school determined to work in semi-demi-hemi-groups, but the only professor who works in the area is Dr. Jerkwad who you do not get along with, then I would be very reluctant to choose him as an advisor. When choosing an area, think about which classes and exams you found most interesting to study for. Usually one can decide, for instance, that they like analysis and topology, but do not care for algebra. Then one should select an advisor from those professors who work in these area(s) and who would also be compatible in other regards. It is sometimes difficult to find someone with whom to work if you have your heart set on a specific area and there are not many professors working in that area. In such a situation it is best to be open minded about the area in which you plan to work. Remember that many subjects are not what they appear on the surface, and techniques from different areas are often applied to them. For example, one may find that doing work in algebraic topology or algebraic number theory will involve as much algebra (or possibly even more) than a pure algebraist would use. Therefore it is a good idea to investigate other areas, talk with people who work in these areas, and find out if you can do the type of mathematics that you like if you choose to work with them.

- Is someone you can learn from.

  People have different styles of learning and doing research. Make sure your advisor is someone who can explain ideas in a way you can understand and who is capable of providing timely, constructive feedback regarding your attempts to do original work. An effective way to determine this is to attend a class or seminar taught by this professor as well as to have individual discussions with them about mathematics.

  Along these lines, you also need to consider how much direction you want. Some advisors will hand you a well-defined thesis-sized problem, explain an approach, and tell you to get to work on it. If you get stuck, they tell you how to proceed. Other advisors are more hands-off; they may give you no help in choosing a topic at all, but can be extremely useful to bounce ideas off of once you find one. You need to think carefully about how much hand-holding you want or need, and make sure to find an advisor who provides that amount. Remember that as you start doing research (particularly if you have never done any before) you will need more guidance, but ultimately you want to evolve into an independent researcher capable of working on your own.

  > *"Some students in the lab are only nominally supervised by a thesis advisor. This can work out well for people who are independent self-starters. It has the advantage that you have only your own neuroses to deal with, not your advisor's as well."*
  >
  > — *How to do research at the MIT AI Lab*

- Has successfully directed students in the past.

  You are new at research; it helps if your advisor has had experience advising graduate students and has done this successfully. If you are thinking of choosing someone as your advisor who has had exactly three students in the past, all of whom dropped out, then you should be very hesitant to work with this person. If a professor has had students in the past who did not graduate or had trouble finding a job, then it is a good idea to try to find out why these students had difficulty before you choose this professor as your advisor.

- Has a reputation as a fair advisor with reasonable expectations.

  You do not want to work with an advisor who never shares credit for ideas, who expects every student to spend 4 years after qualifying exams working on their thesis, or who does not know how to motivate and encourage students. Also find out what kind of expectations this professor has of their students. I have heard stories of professors who will make their students photocopy papers or perform other mundane tasks for them. Some expectations (such as speaking regularly in a seminar) may be appropriate, while others (such as washing their car) may not be as reasonable. In any case, make sure you know what to expect and that you are comfortable with these expectations

- Has a high probability of staying at the school.

  If your advisor moves to another school or goes on sabbatical during your last year and a half in the program, then you will have a severe handicap. You may need to consider moving with your advisor or changing advisors — a difficult situation at best. Even if your advisor goes traveling or is on leave for a term, communication can be difficult and this can affect your work. Along these lines, be careful of choosing an advisor who is coming up for tenure before you graduate. The tenure gods are often fickle, and if this professor does not get tenure, they may leave before you graduate.

- Is someone you like, get along with, and admire.

  You will be working closely with your advisor until your graduation, and the relationship is likely to continue for a long time after that. The more comfortable the two of you are with the relationship, the fewer problems you will have. However, make sure that you do not choose someone solely because you enjoy hanging out with them. Choosing or keeping an advisor primarily because he or she is nice is a mistake. Sometimes nice people withhold frank evaluations of your knowledge, skills, and progress to avoid hurting your feelings. Choose your advisor based on respect rather than fondness.

- Is available.

  Make sure that the person you choose as an advisor has time to spend working with you. You do not want an advisor that has more students than they can handle, or someone who has numerous responsibilities that will take precedence over you. If this happens, your advisor will not be able to give you the time and supervision that you need, and in many ways it will be as though you have no advisor at all. It is important that your advisor does not have too many students. Having a graduate student can be very time consuming, and if your potential advisor is supervising more than a handful of students, you can be sure that this person cannot devote as much time as they would like to each of them. In addition: Beware of the perpetual administrator. If your advisor of choice is someone who has many other professional responsibilities (being an officer in a professional society, being the departmental chair, being the editor of more than one journal, or even applying for tenure), then verify to your own satisfaction that this faculty member will have adequate time to work with you. Professional service activities are important and contribute to the profession of mathematics. Similarly, administrative duties are a necessary evil. However, these things can substantially reduce supervision quality, particularly if you are one of many advisees. Be very careful if you think this is a potential problem with your prospective advisor, and be even more careful if you have heard past students of this person complain of these shortcomings.

Secondary Factors in Choosing an Advisor

- Has an active research group.

  You can learn a tremendous amount from advanced students, and the opportunity to interact with a group of motivated researchers working on similar topics as you is quite stimulating. Be aware, though, that if the group is too big, you may have less time with your advisor.

  In addition, beware of professors who have not published much recently. The best advisor is one who is still actively involved in research. Some older professors may not be as active in research as they once were. Also, some professors may have solved big problems in the past, but currently do not have many problems suitable for students. Look for someone who is enthusiastic about their work and has an active and growing research program, even if this means that you must choose a less experienced or less renowned faculty member.

- Has a national or international reputation among researchers.

  Someday you will finish your degree work and be looking for a job. Your advisor's reputation and professional colleagues could be key in opening doors for you. More immediately, your advisor will be leading your research (at least at the beginning). Therefore, it is important that they are involved in quality work and are studying problems that others will consider important and interesting.

- Has grant support

  Grant support from major research foundations, such as the National Science Foundation, indicates that other scientists judge this person to have made significant contributions. Such grant support is allocated competitively, in fact more competitively than is space in major journals. A history of grant support from major foundations is therefore very impressive. Even more impressive is a researcher who holds a special position where a university or a foundation has granted the person a lifetime of research support.

  At the same time, however, beware that such grants do limit the amount of attention the professor can devote to you because they are busy writing proposals, justifying past work, handling paperwork, or traveling.

If your choice of an advisor does not work out, you may need to change advisors. The earlier in the process you do this, the better. In my own graduate career I saw three people change advisors. Two of them dropped out of school and one did not. The two that dropped out changed advisors after they had been doing research for about a year and were well into their theses. The other, who finished the program, changed advisors early (a few months after she started working with him), and well before she began working on a specific problem. Changing advisors can be a major setback. Often it will result in you starting much of your work over. You will have to learn new topics and get settled into working with someone new. If you must do it, do it early.

In addition, even if you like your advisor you may find that he or she does not always give you all of the mentoring that you need. Multiple mentors are common and useful; they may include other faculty members, older students, or other colleagues. In particular, it can be extremely useful to talk with postdocs and young faculty. They are only a few years out of grad school and have advice which is often timely and applicable.

Chapter 2

Getting grounded in an area

The transition from student to researcher is a time of great upheaval for many as they must simultaneously adjust to additional freedom and additional responsibilities. It is much like the transition from undergraduate to graduate life. There are higher standards of excellence and one is expected to take a more active role in one's own education. Also there are fewer "checkpoints" such as exams and deadlines, and it is easier to yield to an urge to procrastinate. In any case, there are more responsibilities to handle and fewer reference points to rely upon.

After you pass your quals you typically have a lot to learn before you can begin to do research or even begin to look for a project to work on. Having passed a qualifying exam in an area merely means that you know as much as every mathematician should know about this area — not nearly as much as a specialist will need.

As a result, you must spend some time learning the specifics of your area and learning the basics that everyone working in that particular area should be familiar with. The amount of time that this requires varies greatly from area to area. In some areas, such as combinatorics, there is often very little start-up cost and many people begin working on a problem almost immediately. In other areas, there may be a huge start-up cost. If you want to do work in E-theory or algebraic K-theory, it may take you a year or more to get up to the point where you can even understand a problem, much less work on one.

On average, it is reasonable to spend between 6 and 9 months in this stage of reading and gathering information and another 3 to 6 months specializing and choosing a thesis topic. Of course, depending on the area, you may need more or less time. However, if you expect to graduate within 3 years after receiving your Masters degree, 9 to 15 months is a reasonable amount of these 3 years to spend acclimating yourself to the area.

The Necessity of Structure

The key to completing your thesis on time is organization and time management. Sometimes the lack of structure can be difficult as you attempt to plan, carry out, and write up a research project. Structure must be applied throughout this process in order to complete this task. It is easy to think that you are just taking a day or two off now and then only to realize that months have gone by during which you have not accomplished anything. Here is a list of suggestions to keep this from happening to you.

- Come in every day.

  I cannot emphasize the importance of this enough. Every student that I have ever seen graduate has also shown up at the department nearly every day. When you notice that someone has not come in for a while, it is a safe bet that they will soon be dropping out. Visibility is an often underestimated characteristic of successful graduate students. Those who are observed to be physically present in the department stand a much better chance of doing well in the program.

  > "80% *of success is showing up.*"
  >
  > — *Woody Allen*

- Put in a full day's work.

  When first starting to do research, you will have a lot of time on your hands and no immediate deadlines. It is too easy to do just an hour or two of work a day, and think that you are working hard. Some people even go so far as to claim that they can only work for that long because if they read or study more they will not retain it. This is really a cop-out. Granted, you may not be able to read or learn new things for more than a few hours a day, but there are many other tasks that you can do. As you get deeper into research, and particularly as you are finishing your thesis, you will need to put in very long days. It is best to get used to working a full day now, and to do as much as you can now rather than leave it for later. If nothing else, learn skills or do tasks that you will eventually need to anyway: study for a language exam, learn the basics of LaTeX, or look through the current literature to find out what is currently being done in your area (skim the abstracts even if you do not read the papers).

- Meet with your advisor at least once a week (and perhaps even twice).

  This gives you the chance to ask questions when you are stuck and gives you motivation to make regular progress. You should prepare for these meetings. Come to each meeting with a summary of things you have done since the last meeting, a list of questions/topics to discuss, and ideas about what you should be working on next.

- Be organized.

  Take (brief) notes on what you have read to summarize the main ideas and concepts. Also keep and file all papers that you look at. Compile a list of interesting questions that arise as you work.

- Be efficient.

  Only read what you need to. Skim a book or paper before reading it in order to find out if you have the necessary background, to determine which parts are relevant, and to decide how much time is worth spending on it.

- Organize a seminar related to what you are learning.

  This can be particularly useful if there are other students in your area. Also, if you present in the seminar, it can be a wonderful opportunity to organize what you learned in a comprehensible manner.

- Avoid distractions.

  It is easy to ignore your research in favor of more well-defined tasks such as taking classes, teaching, organizing student activities, or writing guides such as this one. Try to minimize these types of activities to only those which are truly necessary. Any work that is not central to making you a better mathematician should be avoided.

Chapter 3

Specializing and beginning a research project

After you have been reading about your area for a while, you will eventually come to a point where you feel saturated by all that you have learned. At this point you should pick an area in which to specialize and begin looking for a problem (or problems) on which to work. You can either ask your advisor for a list of potential areas to specialize in, or you can come up with your own topics and discuss their feasibility with your advisor. Some advisors may even have ideas for specific problems, and your choice of specialization may be influenced by these.

So how do you go about finding a project or problem? There are some aspects of graduate school that are more daunting than others, and finding a research topic is perhaps the biggest obstacle for most students. Sometimes an advisor will initially give you a problem to work on, but ultimately you will need to become self-sufficient and find your own problems. The following are some suggestions on how to do this.

3.1 Finding problems or projects

- Get an idea of what is currently being done in the area.

  In order to conduct original research, you must be aware of ongoing research in your field. Most students spend up to a year reading and studying current research to identify important open problems. However, you will never be able to read everything that might be relevant — and new work is always being published.

  Try to become aware and stay informed of directly related research — but if you see new work that seems to be doing exactly what you are working on, do not panic. It is common for graduate students to see a related piece of work and think that their topic is ruined. If this happens to you, reread the paper several times to get a good understanding of what has been accomplished. Show the paper to your advisor or someone else who is familiar with your topic and whose opinions you respect. You will usually find that the work in the paper is not quite the same, and that there are still directions open to you. You may even end up collaborating with the author. Good researchers welcome the opportunity to interact and collaborate with someone who is interested in the same problems they are.

- Learn to read mathematics papers.

  You will have to read a lot of technical papers to become familiar with any field and to stay current once you have caught up. You may spend a lot of your time reading, especially at the beginning. This is normal. It is also normal to be overwhelmed by the amount of reading you think you "should" do. Try to remember that it is impossible to read everything that might be relevant: instead, read selectively. When you first start reading in a new field, ask your advisor or a colleague for recommendations on the most useful articles in your field, and ask for a list of seminal or "classic" papers that you should definitely read. Also ask for a few names of important or productive researchers currently working in the area.

  Reading papers is often different from reading books. Before bothering to read a paper, make sure it is worth it. Scan the title, then the abstract, then — if you have not completely lost interest already — glance at the introduction and conclusions. Before you try to get all of the nitty gritty details of the paper, skim the whole thing and try to get a feel for the most important points. If it still seems worthwhile and relevant, go back and read the whole paper. Many people find it useful to take notes while they read. Even if you do not go back later and reread them, it helps to focus your attention and forces you to summarize as you read. And if you do need to refresh your memory later, reading your notes is much easier and faster than rereading the entire paper.

  You need to develop the ability to digest and organize vast quantities of information quickly. You will encounter a mind-boggling amount of material. You need to sift through it, select what is worth remembering, and decide what is worth exploring deeper.

- Keep a journal of your research activities and ideas.

  Write down speculations, interesting problems, possible solutions, random ideas, references to look up, and notes on papers you have read. Read back through it periodically. You will notice that the bits of random thoughts start to come together and form a pattern, often turning into a research project or even a thesis topic. Sometimes you will find that similar ideas crop up in your thinking, and that they gradually evolve into a program of research.

- Become an active reader and listener.

  It is very important to make the transition from the passive mode of learning encouraged in traditional lecture courses, to an active and critical learning style that is required to do research. Whenever you read papers or listen to a research talk, ask yourself these canonical questions:

  - From where did the author seem to draw the ideas?
  - What exactly was accomplished by this piece of work?
  - How does this seem to relate to other work in the field?
  - What would be the reasonable next step to build upon this work?
  - What ideas from related fields might be brought to bear upon this work?

  One technique that some find helpful is to keep a written log of your technical reading and listening. Review it periodically to see if some of the ideas begin to fit together.

3.2 Generating ideas

Schedule some time every week for attempting to generate research ideas. Some possible catalysts are:

- Subscribe to the arXiv mailing list in your area. Every few days you will receive a list of titles and abstracts of papers in your area that have been submitted. Read the abstracts and then download and read those papers that seem of interest to you.
- Make a weekly trip to the library to read the abstracts from the premier journals in your field. Choose an article or two to skim.
- Look up papers listed in the bibliographies of books or papers you are currently reading. Read the reviews on MathSciNet and look closer at selected papers.
- Attend a research seminar or colloquium series. Listen and critique.

Take notes on these activities, and ask the canonical questions. As you review your notes 6 months from now, you may find an idea that strikes a chord then but is beyond you now.

Once you have identified a topic that looks feasible, make sure you are aware of all of the literature in the area. If you do not frequently review the literature you read months ago, you may find yourself unconsciously claiming credit for other people's ideas. On the other hand, do not let other people's frame of mind limit your creativity. Continue reading and listening, and keep distinct in your mind what is different between your work and the work of others.

Remain Active

Even after you have decided on your initial focus, it is important to continue a routine of reading new papers and attending seminars. All of these sources can contribute to the development of your idea. At this stage you can add one question to the canonical list: How can these ideas help me solve my research problem? Remember that often the initial idea is quite far from the final thesis topic. If you remain active in reading and listening, it will be much easier to generate alternative topics if you need to.

3.3 Identifying a good thesis topic

The characteristics of an ideal topic are to some extent incompatible. On the one hand, the subject should be timely. Previous groundwork should leave your research problem ripe for completion, and it should be in an active area with potential for future work and employment. On the other hand, you want to work on an open problem that no one else has made significant progress on. If a field is too crowded and the subject too prominent, then you risk being scooped by a more experienced researcher who is able to work faster than you.

A good research topic will address important issues. You should be trying to solve a real problem. Ideally the topic will be related to — but not a simple variation or extension of — existing research. It should be significant yet manageable. Finding the right size problem can be difficult. One effective way of identifying the right size is to read other dissertations. It is also useful to have what is called a “telescoping organization” — a central problem that is solvable and acceptable, with extensions and additions that are successively riskier and that will make the thesis more exciting. That way if the more impressive additions do not pan out, you will still have a solid result.

In the extreme case, if your topic is so out of the ordinary that it is unrelated to anything else, you may have difficulty convincing people it is worthwhile. Truly innovative research is of course exciting, and it may pay back in recognition from the research community — or you could just be out in left field. If you have a far-out topic, be sure that people are actually *interested* in it, or you will never be able to sell it later. In such a situation you may have trouble publishing or finding a job. In addition, it will be hard to find colleagues who can give you advice and feedback on your work.

Sometimes finding a small problem to work on and building on it in a “bottom up” fashion can work equally well, as long as you do not fall into the trap of solving a lot of small unrelated problems that do not lead to a coherent, solid, and substantial piece of research (see Section 4.1).

Remember that a thesis is only a few years of your work, and that — if all goes well — your research career will continue for another 30 or 40 years. Do not be afraid to leave part of the problem for future work, and do not compare yourself to senior researchers who have years of work and many publications to show for it. (On the other hand, if you leave too much for future work, your thesis will not look very exciting.) Graduate students often pick overly ambitious topics. In theory, your advisor should help you to identify a realistic sized problem. At the same time, do not overestimate what other people have done. Learn to read between the lines of grandiose claims in other people's work (which is something a good advisor will also help you do) and find opportunities to make your own contributions.

A good thesis topic will simultaneously express a personal vision and contribute to the existing literature. Your topic must be one you are passionate about. Nothing less will keep you going.

Chapter 4

Conducting research

> *"That's the nature of research — you don't know what the hell you're doing."*
>
> — *'Doc' Edgerton*

It is impossible to explain exactly how research is done, and as the following joke implies, it is beyond the scope of this guide to tell you how to be technically brilliant.

The Feynman Problem Solving Algorithm:

1. Write down the problem.
2. Think very hard.
3. Write down the solution.

However, this guide can provide a general outline of how to organize mathematical research as well as give suggestions on dealing with common difficulties. One should be aware that the skills that make you good at passing classes and exams are often not the same as those that will make you a good researcher. Many people learn this the hard way, and are unwilling to accept it at first. The following are activities that deserve attention.

4.1 Developing a research program

One of the most important tasks that you will have to address when working on your thesis is to create a program of research. You will need to learn to break big problems into smaller ones, outline an approach to proving various results, and organize your work into a cohesive whole. Throughout this process you should try to develop techniques that will not only apply to your problem, but also to other work.

There is a joke that asks "Would you like to know a 24 step program for proving the Riemann Hypothesis?" The answer is to "Count to 23 and then prove the Riemann Hypothesis." There is something to be learned from this. In order to tackle large problems one must learn to form steps and approach them in a manageable way:

- Try a simplified version of the thesis problem first.
- Work examples. Thoroughly explore some concrete instances before attempting to prove something in general.
- Figure out what lemmas or partial results could be used to solve the problem.
- Look for counterexamples. If you cannot find any, try to identify what is preventing their occurrence.

Take a large problem and create a few smaller problems on which to work. If you cannot make any progress on one, try to work on another and return to it later. Do not be worried if at first you cannot seem to make any headway or if you are confused about how to approach the problem. Keep trying and eventually you will make some progress.

> *"Research is what I'm doing when I don't know what I'm doing."*
>
> — *Wernher von Braun*

In addition, be sure to work on a variety of problems and investigate general questions. Be careful of putting all your eggs in one basket. You do not want to say, "I am going to prove a quasi-contravariant global chromatic embedding theorem for manifolds of dimension four" and work exclusively on that for three years — particularly if you are not making progress. Since this is a very specific problem, you would be well advised to look at related questions. By all means work on this particular problem, but also consider additional questions and work on them as well. For example, if you are working on the problem stated above, you could try to think of some other interesting questions about embedding four-manifolds that you can work on simultaneously. By working on multiple problems or considering general questions you have a greater chance of success, and you also will be in better shape if a counterexample is found to one of the conjectures you are trying to prove.

> *"The outcome of any serious research can only be to make two questions grow where only one grew before."*
>
> — *Thorstein Veblen*

4.2 Getting stuck and dealing with failure

"I have not failed. I've just found 10,000 ways that won't work."

— Thomas Edison

All research involves risk. What is hard is dealing with failure. It is easy to interpret your project failing as you failing. In reality it proves that you had the courage to try something difficult.

The few people in the field who consistently turn out papers year after year actually fail as often as anyone else. You will find that they often work on several projects at once, only a few of which pan out. The projects that do succeed have usually failed repeatedly, and many wrong approaches have preceded the final success.

As you progress through your career, you will accumulate a lot of failures. But each represents a lot of work you did on various subtasks of the overall project. You will find that a lot of the ideas you had, ways of thinking you developed, and often even lemmas you have proven, turn out to be just what is needed to solve a completely different problem several years later. This effect only becomes obvious after you have piled up quite a stack of failures. So take it on faith as you collect your first few that they will be useful later.

Research always takes much, much longer than it seems it ought to. The informal rule is that any given subtask will take three times as long as you expect. (Some add, "even after taking this rule into account.")

Crucial to success is making your research part of your everyday life. Most breakthroughs occur while you are in the shower, walking in from the parking lot, or sitting on your couch daydreaming. If you are thinking about your research in background mode all the time, ideas will just pop out. Successful mathematicians are generally less brilliant than they are persistent. Also very important is developing "taste", the ability to differentiate between superficially appealing ideas and genuinely important ones.

You will find that your rate of progress seems to vary wildly. Sometimes you go on a roll and get as much done in a week as you had in the previous three months. That is exhilarating; it is what keeps people in the field. At other times you get stuck and feel like you cannot do anything for a long time. This can be hard to cope with. You may feel like you will never do anything worthwhile again; or, near the beginning, that you do not have what it takes to be a researcher. These feelings are normal and almost certainly wrong. You need to hang in there, maintaining high tolerance for low results.

You can get a lot more work done if you start setting short and medium term goals — weekly and monthly, for instance. In this way you can try accomplish smaller, intermediate tasks and keep track of your progress. Nonetheless, you will get completely stuck sometimes. Like writer's block, there are many causes and no one solution.

Setting your sights too high often leads to paralysis. Work on a subproblem to get back into the flow. Do not get into a loop in which doubts about your ability to do the work eat away at your enthusiasm so that in fact you cannot get anything done. Realize that research ability is a learned skill, not innate genius.

Fear of failure can also make it difficult to work. If you find yourself inexplicably "unable" to get work done, then ask whether you are avoiding putting your ideas to the test. The prospect of discovering that your last several months of work have been for naught may be what is stopping you. There is no way to avoid this; just realize that failure and wasted work are part of the process.

Most people find that their personal life and their ability to do research interact. For some, work is a refuge when everything else is going poorly. Others find themselves paralyzed at work when life is in turmoil for other reasons.

In addition, there is a well-understood phenomenon known as "thesis avoidance," whereby you suddenly find yourself working on mundane tasks or fiddling with minute details and convincing yourself these are utterly fascinating or of paramount importance. This is invariably a subconscious way of getting out of working on your thesis. Be aware of what you are doing. (Some could argue this document is itself an example of thesis avoidance on the part of its author.)

There are a number ways you can waste a lot of time while working on your thesis. Some activities to avoid (unless they are necessary for your work): doing unnecessary preparation for a class, surfing the web, playing games, emailing for long periods of time, and excessive socializing at the office. Any work that is not directly contributing to the completion of your thesis should be minimized.

4.3 Developing tenacity

> *"Let me tell you the secret that has led me to my goal. My strength lies solely in my tenacity."*
>
> — *Louis Pasteur*

You do not have to be a genius to do well in graduate school. You must be reasonably intelligent, but after a certain point other traits become more important in determining success. Nobody finishes a dissertation without being tenacious. A dissertation usually takes a few years to complete. This can be a culture shock to former undergraduates who have never worked on a project that lasted longer than one quarter or semester (at the end of which, whatever the state of the project, one declares victory and goes home). No one can tell you in advance exactly how long the dissertation will take, so it is hard to see where the "end of the road" lies. You will encounter unexpected problems and obstacles that can add months or years to the project. It is very easy to become depressed and unmotivated about continuing. If you are not tenacious about working on the dissertation, you will not finish.

> *"Genius is nothing but a great aptitude for patience."*
>
> — *George-Louis De Buffon*

The best way to finish a thesis is to do something every day that gets you closer. Do not work on it only when you are in the mood or feeling productive. Be disciplined and keep going through both the good times and the bad. This will ensure that you finish.

> *"The Germans have aptly called Sitzfleisch the ability to spend endless hours at a desk doing grueling work. Sitzfleisch is considered by mathematicians to be a better gauge of success than any of the attractive definitions of talent with which psychologists regale us from time to time."*
>
> — *Gian-Carlo Rota*

Chapter 5

Becoming part of the research community

One of the most important things that you, as graduate student, must accomplish is to establish yourself as part of the research community. Your advisor can help with this process by funding conference travel, encouraging you to publish research results as soon as possible, collaborating on joint publications, introducing you to colleagues, and promoting your work.

In turn, you can make yourself more visible by participating in conferences and workshops, publishing papers on your work, talking to others about your research, meeting new people in your research area, and maintaining contact with colleagues at other institutions.

5.1 Attending conferences

Attending conferences and workshops is valuable whether you give a talk or not. Some of the reasons to do so are:

- You will meet people and have a chance to discuss your ideas and to hear theirs.
- You will gain a sense of the current state of research and will learn more about how to write papers and give talks. (Unfortunately, you may sometimes learn this by counterexample.)
- You will probably realize that your ideas are more significant than you thought. You may see a talk and think "I could prove results like this!"

If you are giving a talk you will gain even more visibility, and will have an opportunity to make an impression on other researchers. Here are some tips for preparing your talk to make a positive impression:

- Give a practice talk, especially if you tend to get stage fright. Be sure to invite people who will give you constructive and useful feedback.
- Make sure your talk fits in the time slot allocated. There is nothing worse than a speaker who rushes through the last ten slides or skips from the middle of the talk to the conclusion. A good rule is to allocate 2-3 minutes per slide.
- It is better to be somewhat vague than to get bogged down in technical details. However, be sure you give enough detail so that people understand what you're saying and believe your conclusions. The purpose of a talk is to convey the main idea of what you're working on. Make your preprints available, so that people can read it to fill in the missing details or obtain a more in-depth understanding.
- Know your audience: for a general audience you will need to provide more background and motivation; while for an audience of specialists you may have to provide more technical details.
- Use examples and pictures to illustrate and clarify your ideas.
- Learn by observation: try to imitate qualities of talks that you like, and avoid those aspects that bother you.
- Talk about your ideas informally whenever you get the chance, so that the presentation will come more naturally. This also gives you the chance to hear other responses, and anticipate questions that may be asked at your talk.
- Make sure your slides are as readable and as simple as possible. Never put up a slide with tiny text and say "I know you can't read this, but . . ."
- Try to relax. Do not read from a script or word-for-word from your slides. Do not talk too fast. Be confident: you know more about your work (flaws and all) than anyone else.

Just going to conferences and standing in the corner is not enough. If you are not an outgoing person, you have to make a conscious effort to meet and build relationships with other researchers. Presenting papers is a good way to accomplish this as people will often approach you to discuss your presentation. Introducing yourself to people whose presentations you found interesting and asking a relevant question is also an effective way to meet people. Sometimes it is easier to initiate conversations with other graduate students than senior researchers — this is also productive as those graduate students may provide contacts to the senior people they know, and someday they will be senior people themselves.

You should talk about your research interests every chance you get. (But be sure to spend some time listening, too.) Have summaries of your work of various lengths and levels of detail mentally prepared. That way you can answer the inevitable "So what are you working on?" intelligently and clearly. If someone expresses an interest in your work, then follow up. Send them email talking about new ideas or asking questions; send them drafts of papers; ask them for drafts of their papers and send them comments. (If you do this in a polite and helpful way, they will be sure to remember you.)

Maintain the relationships you form, either through email or by re-establishing contact at each conference you attend. If you work at it, and use your initial acquaintances to meet new people, you will find that your network grows rapidly.

Sometimes these contacts will grow into opportunities to do collaborative research. Seize these opportunities. They will allow you to meet more people and be exposed to new methods of doing research or new subfields within your area. In addition, the responsibility you feel towards your collaborator may give you more of an incentive to stay motivated and keep accomplishing something.

5.2 Publishing papers

5.2.1 Why we publish

Publishing your ideas is important for several reasons. Although the primary purpose for publishing your work is to contribute to the mathematical community, it also has many benefits for you as the author. It gives you a source of feedback from those who read your work; it establishes you as a member of the research community (useful for getting a job down the line); and it forces you to clarify your ideas and to fit them into the context of the current state of research in your field.

5.2.2 What to publish

There are three necessary conditions for a paper to be published. The results must be

1. Original
2. Interesting
3. Correct

In addition, the important ideas should be well developed and the writing style should be clear.

If you have a great idea but present it poorly, your paper will probably not be accepted. Be sure you know what the point of the paper is, and state it clearly and repeatedly. The same goes for the key technical ideas. Do not make the reader work to figure out what is important — tell them explicitly. Otherwise, they might get it wrong; that is, if they bother to finish reading the paper at all. State the problem you are addressing, why it is important, and how you have solved it. Be sure to also describe the results you have, how other researchers have addressed the same or similar problems, and why your method is different or better.

Write for the audience that you expect to read the paper. Give more background for general audiences and more technical details for specialized audiences. Use a running example, if possible, especially if your paper is dense with equations and notation. In addition, do not try to put every idea in your thesis into one paper. Break it down into pieces and write two or more shorter journal articles.

It is critical (at least at the beginning of your career) that you have someone else proofread any paper before you submit it. At the very least, this other person can check for typos, look for grammatical errors, and give you comments on style. Once you've submitted a paper, a good reviewer will give you feedback on the organization and content of the paper as well. (Of course, at this point that journal will have already decided whether to accept or decline your submission.)

If your paper is rejected, keep trying. Take the referee's comments to heart and try to rewrite the paper. After reading the referee's report for the first time, you may want to put it aside. After you've had some time to think, come back to the paper, read the referee's report again, and decide whether the criticisms were valid and how best to address them.

You will often find that reviewers make criticisms that are off-target because they misinterpreted some aspect of your paper. If so, do not let it get to you — just rewrite that part of your paper more clearly so that the same misunderstanding will not happen again. It is frustrating to have a paper rejected because of a misunderstanding, but at least it is something you can fix. On the other hand, criticisms of the content of the paper may require more substantial revisions such as rethinking your ideas, simplifying arguments, or fixing a proof.

5.2.3 Where to publish

Choosing where to send your work for publication is an important decision. Of course, certain material is more suited for some journals than others. However, there are many more subjective concerns that one must take into account (e.g., How good is it?, Will others be interested?). Submitting your work is often a gamble. On the one hand, you want it to be published in a prestigious journal. But on the other hand, you do not want to have a paper rejected too many times. Since submitting papers is often a very lengthy process, any rejection will cause your work's appearance to be delayed by months or even years. This is not a good thing for someone who is applying for jobs or up for tenure.

Hence you must gamble. Try to choose a publication that is as prestigious as possible, yet for which you also think you have a good chance of getting accepted. At the same time, try to choose a publication for which the paper is best suited. The following are some issues one should take into account.

- The subject of your paper

  You should first find out which journals are appropriate for the particular area and topics in which you work. If you work with Fourier transforms, then an Analysis journal would probably be appropriate. However, if your work involves Fourier transforms on finite groups, then an algebra or applied math journal may be even more appropriate. Learn what kinds of journals publish work done by yourself and others in your area. One good way to do this is to look at the bibliography of your paper or other papers that you frequently use, and see where those articles were published. Another excellent way to find out about journals is to talk to your advisor or others who work in your area (at conferences, for example) and ask them their opinions on various journals. Often these people can tell you about experiences they have had with particular journals as well as give you some inside scoops. For example, you may find that a journal prefers to publish articles of a certain kind even if they do not advertise themselves in that way.

- The length of your paper

  Another important factor in choosing where to send your paper is its length. Lengths of papers vary greatly between areas so it is difficult to say what constitutes long papers versus short ones. Most journals (usually on the first page or on the inside cover of each issue) list the lengths of articles they typically accept. If a journal says that they publish articles between 10 and 20 pages in length, and if you have a 30 page paper, then it is best you submit it elsewhere. Unless your work is truly outstanding, it will almost surely be rejected.

- The quality of your paper

  This is the most subjective aspect of deciding where to publish. You need to have some sense of how good your work is, and how good others will think it is. You probably have some idea of what you think of your results, either by comparing it to your other work or to the work of others. You can get a sense of what others think by circulating preprints or asking others to proofread your paper.

  In addition, you need to gain a sense of how prestigious the various journals are. The best (perhaps only) way to do this is to talk to your advisor and others in your field. You will find that although *The Journal of Operator Theory* and *The Journal of Functional Analysis* have similar titles, acceptance in the latter is much more difficult and hence more prestigious. Often people may disagree on exactly which of two journals is the better, but if you talk to enough people you will eventually gain a sense of which journals are consistently thought best, which are middle of the road, and which are dumping grounds for mathematical minutiae.

Another way to determine the relative prestige of journals is to look at their "Impact Factor". Every year each science journal is assigned a number, called the Impact Factor, that reflects how often the papers in it are cited — the higher the impact factor, the more often the papers published by that journal are cited in other's work. People sometimes use the Impact Factor to compare the relative merits of two journals and decide which is more prestigious. This is extremely controversial. Many people argue that the impact factor does not correlate with quality, and many people also argue that the Impact Factors assigned to many mathematics journals do not correspond with how they are viewed by the mathematics community (so that, for example, a mathematics journal viewed as prestigious in the mathematics community may have a lower Impact Factor than another mathematics journal that is viewed as not as good). Also, Impact Factors are notoriously bad at comparing journals across disciplines; mathematics journals, for instance, usually have Impact Factors between 0 and 2.5, whereas journals in biology or medical sciences can often have Impact Factors over 20.

Impact Factors are proprietary information, so in order to have access to them, your library needs to subscribe to a service. Libraries at most large colleges and universities tend to subscribe, and if you are at such a school you can get access. Alternatively, you can ask your colleagues (some of whom may be at universities that subscribe) if they have access and can send you a copy.

Despite the controversy surrounding Impact Factors and despite the debate surrounding their limitations in measuring quality of work, many universities use Impact Factors when deciding tenure, promotion, and retention. So it is a good idea to be aware of the Impact Factors of journals to which you submit your papers, and remember that your college or university may be using them to evaluate you.

- Amount of time

  A mathematician who will soon be on the job market or up for tenure is often concerned about getting their work published (or at least accepted) as soon as possible. Some journals take much longer to publish papers or even to respond to submissions than do others. The AMS publishes an annual backlog of the major journals (available in the *Notices* as well as on the web) which gives the average length of time for articles to appear. Of course, regardless of where you send your paper, you are still taking your chances with the referee. But hopefully a good editor will make it their job to follow up and ensure that the referee does not take unusually long.

  Finally, a word on the topic of electronic journals. While almost all journals have electronic versions, there are some (often very new) journals which are published exclusively on the web. *The New York Journal of Mathematics* is one such example. Electronic journals have the advantage that they are able to publish much more quickly and cheaply than paper journals. Consequently, it often takes less time for a paper to appear in print — in fact once a paper is accepted it may be published in a few days, rather than the typical months or years that conventional journals take. On the other hand, journals that are exclusively electronic are often fairly new and do not have the reputation of the older, more established journals. Therefore they are not always the best place for a beginning researcher to publish.

5.2.4 How to publish

Once you have decided what journal to send your paper to, you will need to submit it. Here is a rough outline of the steps you will follow.[1]

1. Send the Paper.

   Once you have decided where to send the paper, find out the requirements for submission. You can find these either on the inside cover of a copy of the journal or at the journal web site. Begin by making sure that your article fits all of these requirements (e.g., the correct length, the right kind of material). If not, then you will have to choose another journal to submit your paper to.

   Next choose an editor to whom you will send your paper. Again, there will be a list of editors on the inside cover of the journal or at the journal web site. You should be very careful about which editor you choose. Ask others whether this person is known to be a responsible and organized editor who follows up on submissions. Some editors develop reputations as "black holes" — you send a paper to them and never hear from them again, sometimes to the point that they will not even respond to mailings that you send them. If you talk to people in your area, you will almost always hear of editors like this. Avoid them at all costs.

   After you have chosen an editor, look at the requirements and requests regarding submissions. Typically, you will send an email to your chosen editor with a short message and a copy of your article in PDF format.

   Some journals may request that you use special sized margins, or even that you download a TeX style file from their web page. Do not worry about these things until after your paper has been accepted. Sometime they will even make these changes for you, and you do not need to worry about it.

[1] You should be aware that you can only send your paper to one journal at a time. When you submit a paper you are giving the journal the right to publish it if they so choose and are implicitly stating that you have not sent it elsewhere. Submitting mathematics papers is not like trying to get a novel published. You cannot take a shotgun approach and send it to hundreds of places simultaneously hoping that one will accept it. You must submit it to one journal at a time.

Make sure that the copies of your paper have your name, address (both email and postal mail), and the appropriate AMS category (available from the AMS web site). Also, it is usually best to send a PDF file of your paper for the initial submission. If you send the TeX or LaTeX source it may not compile correctly on their machine, and if you send a DVI file it may be unreadable if the recipient does not have the proper fonts. Later, if the paper is accepted, the editor will ask you for a copy of the TeX or LaTeX file.

With your submission you should include a short letter to the editor. Do not just send copies of an article to the editor without specifying which journal you are submitting to. Some people may be the editor for more than one journal. Or worse yet, they may think you are just some mathematician sending them preprints of your work, and not realize the papers are for submission. Your letter should be short (one or two sentences is fine) and should state the title of your article and the journal you are submitting it to. The canonical example is:

Dear Professor Well-known,

I am submitting a copy of my paper *Classification theorems for perverse sheaves generated by aberrant rings* to be considered for publication in the journal *Rings and Things*. Thank you for your time.

Sincerely,

Me

If sending an email, it might also be a good idea to include the name of your institution and your postal address in your signature.

2. Wait.

Shortly after you've submitted your paper, the editor will typically send you a letter confirming that he or she received it. The editor will then send the paper to a referee who will (eventually) read it and give a recommendation as to whether the paper should be accepted or rejected. The identity of the referee is kept secret from you, so that their response can be candid. After the referee has read the paper and made their decision, they will write a report critiquing the paper and explaining their decision. They then send this report to the editor, who will forward it on to you. Ultimately it is the editor's decision whether to accept the paper or not, but usually they will do what the referee recommends. If you have submitted a paper and have heard nothing after 6 months or a year, you should definitely contact the editor and follow up on the paper's status.

3. Respond to Referee's Report.

   Eventually you will receive the referee's report. There are basically three answers that you could get.

   (a) The paper is accepted as it stands.

       This is the best possible response. At this point the editor will probably request a TeX or LaTeX file of your paper. The editor may also be able to tell you the issue in which the article will appear.

   (b) The paper is accepted modulo revisions.

       Here the acceptance is contingent on you making certain changes. Typically these revisions are reasonable, but other times they may be difficult (for instance, if you are asked to shorten the paper by half). You need to decide whether you want to make the changes or not. If not, then inform the editor and submit it to another journal. If you are willing to make the changes, then do so quickly and return the paper to the editor. Again, the editor may ask for a TeX or LaTeX file of your paper. They will then tell you if the changes are acceptable and if the paper is now accepted.

   (c) The paper is rejected.

       In this case you need to choose another journal and begin again at step one. Remember that even the best papers are sometimes rejected, and every mathematician has had papers rejected. Also be aware that papers are rejected for a variety of reasons, not all of which have to do with the quality of the paper under consideration. (For example, journals with large backlogs will sometimes impose higher than usual standards and tell referees to only recommend exceptional papers for publication.) A good referee will provide constructive comments on how to revise your paper to make it better. If you have a paper that has been rejected a few times, try rewriting the introduction. Make sure that it is clear what you are doing and why it is important.

4. Proofread the Galley.

   After your paper is submitted the journal will typeset it and send you a galley. The galley shows how the paper will appear in print. You need to proofread it to make sure everything looks okay, and fix any mistakes that were created in the typesetting process. You then send these corrections back to the editor.

5. Eagerly await seeing your article in print.

   When the article appears the journal will typically send you reprints that you can proudly circulate to colleagues, friends, and family.

Be aware that this process can take a long time. The time from submission to acceptance to appearance can often take more than two years.

5.3 Informing others of your work

Since it often takes years from the time a paper is written until the time it appears in a journal, you will want to inform others of your work as soon as you have a preprint ready. The following are a few ways that you can make others aware of your work.

- The arXiv preprint server.

  The arXiv preprint server (originally hosted by Los Alamos National Labs and formerly called the "LANL preprint server") is used by numerous mathematicians in all areas of mathematics. The easiest way to access the arXiv is through the front at http://front.math.ucdavis.edu/, although it can also be accessed at http://arxiv.org/ or http://xxx.lanl.gov/. At the arXiv you can upload copies of your preprints so that anyone can see them and download copies — all you need is a web browser and a copy of your TeX file. You can also search for, view, and download other people's preprints. In addition, you can subscribe to receive preprint updates by email. You simply specify which category (e.g., Operator Algebras, Algebraic Topology, Number Theory) you would like to subscribe to and then every few days you will receive an email listing all of the preprints that have been posted in this category since the last mailing, as well as all papers in this category that have been updated. (This is usually between 1 and 10 papers). Since many mathematicians use the arXiv, this is a wonderful way to stay informed of what is currently being done in your area. In fact, it is often easier and more informative than watching for papers as they appear in journals — although you still have to keep an eye on the journals to make sure you do not miss anything. If you are not currently on the arXiv mailing list, I suggest that you go to the web site immediately and subscribe to receive updates in your area.

- Create a web page

  A web page is an extremely effective way of making your papers available to others. As soon as you begin your research you should create a web page, and as soon as you have preprints you should make them available on your web site. Your web site does not have to be fancy. It is enough to have your name, your contact information, and copies of your papers available for download. If you know how to write HTML it takes about ten minutes to create a simple web page. If you have never used HTML before, simply ask another grad student or your advisor to show you what to do. They should be able to teach you the basics in about twenty minutes. It is really quite easy. When you are finished creating your web page, contact the department webmaster and ask them to create a link to it from the department web page where grad students are listed. It is important to have your university link to your page because many search engines, such as Google, will rank pages based on the importance of the organizations that link to them. If a high profile place such as a university links to your page, then it will be listed much earlier when people do a search for your name or the titles of your papers. (There are also various HTML tricks such as the use of Metatags that can help your site to be listed higher on internet searches. Consult a book on HTML or ask another grad student to find out more about these.) You will be surprised at how many people will look at the papers on your web site. Furthermore, when it comes time to look for a job, you will find that a web page is also useful to potential employers.

- Send your preprints to people

  This is a more aggressive tactic. It is used frequently by people early in their career and much less by more established mathematicians. If you know of people who are working on topics related to yours or who may be interested in your work, then by all means send them copies of your preprints. You can either send these by U.S. mail or email, and include a brief letter introducing yourself and saying that any comments they have would be greatly appreciated. Also be sure to include your contact information in your letter. Most people will not read your papers, but some people will send you copies of their preprints and one or two may actually have comments for you. More importantly, however, almost everyone will read your name and remember that you sent them preprints. This is a wonderful way to get your name out there and inform people of your presence even if they do not take the time to read your papers. Later when you go to conferences and meet these people it will also serve as an icebreaker. When they see your nametag they will often say something like, "Oh, you're the person that sent me those preprints. Thanks."

Chapter 6

Writing up results

The process of writing up results, particularly in thesis form, takes much longer than many students originally anticipate. There are several reasons for this.

- Writing results in a form that other people can understand is a very slow process.
- In order to produce a well-written paper, many drafts must be written and often large portions of preliminary writing must be completely discarded.
- In the course of your several years of research, you have probably changed notation several times, developed new points of view on your work, and developed many results that looked significant at the time but now seem to contribute nothing toward your final product. Sorting through all of your work and reorganizing it is a lengthy process.
- Even if you have written papers discussing partial results, the audience for your thesis is different, and consequently the style of exposition must be changed significantly. A research paper is addressed to a group of experts in the field, who presumably know the literature and the background issues quite well. A thesis is written more for the general mathematician. A thorough literature review must be included, as well as an evaluation of where your work fits into the scheme of things.
- All the small details that were put off and forgotten must now be included. Citations must be checked, the historical progression of various results must be carefully documented, and the trivial cases must be worked through.
- Your thesis advisor will probably have strong opinions on how the work should be presented. Adapting your style to these requirements may take some flexibility and thought.
- When your advisor or other initial readers first look at your thesis, they will often find undefined terms or symbols, holes in your arguments (or at least in your presentation of them), and other deficiencies.

6.1 Minimizing the pain of writing

Some habits begun early in your research will help:

1. Keep careful notes about your work. You might choose to keep a notebook of results and thoughts of how they will fit into your thesis.

2. Keep track of your results and your thoughts: Carefully write down every proof you create and check it carefully. Also, make an entry at least weekly in your notes even if you believe that you have accomplished nothing of significance. Simply recording what you are thinking about can be helpful.

3. If possible, type up each piece of the work (whether for publication or not) as it is completed. This makes the final writing easier because the proofs are checked and your thoughts and comments are recorded at the time of their completion rather than months or years later. Furthermore, the early write-ups give a basis for organizing your thesis.

4. As you read papers and published works, be a student of technical writing styles. Pay attention what works and what does not.

6.2 Organizing the thesis outline

An important early step in the thesis is to develop a tentative outline. The outline will probably change several times, but it is important always to have a current one foremost in your mind so that you can make the pieces fit together smoothly.

A typical outline will be of the form:

Chapter 1: Introduction

- What is the problem?
- Why is it important?
- What have other people done?
- What is the central idea of your approach?
- How is the rest of the thesis organized?

Chapter 2: The problem

- Define the problem.
- Introduce definitions and terminology.
- Discuss the basic properties.

Chapter 3: Big idea 1

$\vdots$

Chapter k+2: Big idea k

$\vdots$

Final Chapter: Conclusion

- Recapitulate what has been accomplished.
- Present some examples and work through them in detail.
- Discuss ideas for future work.

Do not think that the thesis must be written starting at page 1 and continuing until the end. Most often, the presentation of the big ideas shapes the presentation of the problem. The introduction is often written (or at least rewritten) last. The important thing is to jump in and begin writing something. Make notes along the way as to how other sections need to be adapted so that they all work together.

Remember that the style of thesis writing is expository: you are trying to communicate your ideas, explain the significance of your work, and discuss the limitations of your results. It is not the compressed style of a page-limited paper or journal article. Do not make your reader work too hard. It is easy to be fooled into thinking that since something is now obvious to you after several years of study, it is also obvious to your reader. The most difficult part of thesis writing is organizing and presenting your material in an understandable way.

6.3 Fulfilling university thesis requirements

Most universities have a set of style requirements for theses. Typically these requirements give rules for the appearance of the title page, the width of the margins, the numbering of pages, etc. Often you can get a copy of a LaTeX template from students who came before you so that you can satisfy these rules easily.

6.4 Remain active while writing your thesis

When it comes time to write up your work into thesis form, remember to stay active in other research activities while you are writing. It is an all too common occurrence to see a student in their last year of graduate school retreat from seminars, classes, and other mathematical interactions in order to cloister themselves in their office or at home and type up their thesis. Withdrawing in this way can be very detrimental to your future success. Keep in mind that your thesis is just one step (ideally an initial step) in a long career of scholarship. You should therefore think of your thesis as a beginning — rather than an end — to your research. When you write up your thesis, and also when you write up papers, you want to continue looking for future problems to work on, explore new ideas you may have, and keep up with the work of others. If you've completely solved some problem in your thesis, then you will want to find a new direction in which to move in the future. And if not, you will want to find other ways to continue the project you have underway. In either case, you need to keep working and continue to be part of the research community. The last thing you want is to finish writing up your thesis and graduate only to realize that you haven't done any new work in months and now have no idea of what to do next. You need to keep your research program alive, and ideally let it evolve in new directions. Therefore, when writing up your thesis you should continue with your usual activities: come in to the department regularly, attend and speak in seminars, try to prove new results, read papers, and discuss your work with others. Even if you don't intend to do any research whatsoever after you graduate, you still have a responsibility to continue participating in the department community until you do graduate. If you stop interacting and suddenly disappear to work only on your write-up, it is very noticeable to everyone in the department — especially to your advisor and those people who are going to be writing letters of recommendation for you when you apply for jobs. You want them to be able to tell others that you are a student who is transitioning into the role of a promising scholar, not someone who has just finished the requirements for a Ph.D. and has no future plans.

6.5 Writing clearly and carefully

> *"Wordsmithing is a much greater percentage of what I am supposed to be doing in life than I would have ever thought."*
>
> — *Donald E. Knuth*

As Knuth has commented, wordsmithing is a much larger part of being a mathematician than most people realize. A significant part of the research process involves writing up your results in an accessible and understandable way. In addition to proving theorems, any mathematical work will contain exposition that should motivate the results, put the them into a greater mathematical context, and explain intuitively what they mean. This is an extremely important skill, as anyone who has read a poorly written paper can confirm. It is a shame that more mathematicians are not better trained in the art of writing mathematics.

Here is a brief description of the process I go through when writing up results. Afterward are some comments specific to writing a thesis and specific to writing a paper.

Step 1: Initial handwritten work

Initially you are just trying to get ideas and write down anything that will help you solve the problem. After you think you have a way to prove something take out a fresh sheet of paper and write down the proof. At this point notes are made, work is crossed out and corrected, and there is little worry about choice of notation. You are simply trying to write out a mathematically correct proof. If you make too many notes or cross out too much, you may wish to iterate this step until you have a clear handwritten version of a correct proof.

Step 2: TEX up statements of your results and proofs

Carefully decide on any definitions or notation that you wish to use, and then TEX up your results. At this stage the document will often take the form of lemma, proof; lemma, proof; theorem, proof; ... with an occasional definition or comment thrown in. Throughout this write-up you should check your proofs a second time for validity.

Step 3: Write a rough draft

Write the expository material that will accompany your theorems and proofs. Put in any preliminary material that is necessary for the reader as well as citations and references. Attempt to clean up your notation and definitions. Also try to to simplify your proofs or reorganize them in a more efficient manner. For example, ask yourself if one lemma could replace much of the work you did in three different proofs. Make sure that you write clearly and understandably and that you do not use any new terms without first defining them.

Step 4: Write an introduction and abstract

Because the project on which you are working will evolve in ways that you cannot imagine, it is best to leave the introduction and abstract until after you have written the first draft. The introduction should begin by motivating the problem you are considering, and should conclude by stating the results obtained and their significance. In addition, you may want to briefly explain the techniques used or discuss related results. Remember that a mathematics paper is not a mystery story. You do not want to prove several unmotivated lemmas, and then suddenly unveil some result at the end. State up front what it is that you have done. However, you should feel free to avoid explaining any technical hypotheses or requirements for the theorem. Give a statement of your results in clear and easy-to-understand terms. Perhaps you may want to state a special case of the theorem in which the result takes a particularly nice form. Or you may want to present and examine an important example. In any case, you need to explain what is done in the paper as well as convince the reader that it is interesting and important. It is fair to say that in an introduction you are trying to “sell” the paper.

After this you should write an abstract for the paper. An abstract should be a few sentences summarizing the paper's results. Again, this is often easiest to do after you have a rough draft of everything else. Be aware that some journals do not allow you to place citations in the abstract, so you should try to avoid this if possible.

The abstract and introduction are in some ways the most important part of the paper. They can greatly influence both a reader's and a referee's understanding of your work. Furthermore, it has been said that of the people who read a paper 80% will read only the abstract, and 80% of those that continue will read only the introduction. Since many people will only encounter these portions of your paper, you should write them with care.

A final note about titles: Choose a title which is descriptive but not too long. Avoid using any new terminology in the title. For example, if you are writing about A–B-equivalence, which is defined only in your paper, then do NOT title your work *Classification of A–B-equivalence*. The title should be understandable by someone who has not yet read your paper. In addition, avoid titles which are extremely vague. Titles such as *Compact Hausdorff spaces* or *Operators on Hilbert space* are extremely nondescript since there is much known about both of these topics. Finally, do not use unnecessary phrases. Papers such as *Some results on quantum groups of spheres* or *Concerning quantum groups of spheres* could just as well be titled *Quantum groups of spheres*.

Step 5: Proofread

At this step one should proofread the paper (no pun intended). Check again to make sure that statements contain all of the necessary hypotheses, that your proofs and arguments are correct, and that the exposition flows smoothly. Continue revising the paper until you have met or exceeded your own standards for clear mathematical writing.

Step 6: Give the paper to someone else to read

Have your advisor or colleagues read through your paper. They will give you suggestions for improvements as well as feedback on what parts of the paper were confusing. Ideally, they will also be able to identify any gaps in proofs that may exist. If you go through multiple revisions of a paper, do not expect the same person (even—perhaps especially—your advisor) to keep reading new drafts. You should only give a revised draft to your advisor or another reviewer if the paper has changed substantially and they have said that they are willing to reread it. Be aware that it is not only a great inconvenience for someone to reread a paper, but after the first time they will unavoidably end up skimming much of it.

Step 7: Final Rewrite

While others are reading you paper, set it aside and work on something else. When they are finished, listen to their suggestions and make changes based on them. Then read through the paper one last time. Remember that you will never find all the typos and small errors, so do not be disheartened if you find some later.

> *"Dissertations are not finished. They are abandoned."*
>
> — *Fred Brooks*

Some notes specific to theses: Remember that your thesis can be as long as you want. Feel free to take the time to develop the material and explain it as much as you like. Put in some historical background, provide some examples, or consider special cases of the theorems. A thesis generally serves two purposes. First, and most importantly, it provides you with a record of the work you have done so that later in your mathematical career you can look back at these results and remind yourself of the various details. The second purpose is that it can function as a starting point for future students who are going to work in the same area. Write your thesis with these two audiences in mind.

Some notes specific to papers: When writing a paper be sure to include your AMS category at the bottom of the first page. A list of these categories can be found on the AMS web site. Also be sure to include your email and postal mail address.

In addition, make sure to update the bibliography. It can take a long time for a paper to get accepted for publication. In that time preprints listed in your bibliography may have been accepted and/or published. You need to make sure that this information is updated. Also make sure that any names listed in your paper are spelled correctly. Some names are notoriously hard to spell. Once in a paper of mine I misspelled the name Wojciech Szymański as Wojciech Syzmański. I later received an email from Dr. Szymański informing me of my mistake. This is an embarrassment you would do best to avoid.

Unlike a thesis, when writing a paper you need to be concerned about its length. A paper that is very long is difficult to get published. You need to keep it short and include only what is necessary. If there are many details or long calculations that you need to work out, you may wish to write two versions: a longer version for yourself with everything worked out in gory detail, and a shorter version to be submitted for publication.

6.6 Avoiding pretension, allusions, and flippancy

When writing mathematics your goal is to convey information to the reader. Therefore you should strive to communicate in an efficient and straightforward manner. Use simple, declarative sentences and write with reader in mind.

Avoid pretension in your writing. Do not use big words when small ones will suffice. Be careful of referring to certain facts as "trivial" or "easy to see", and when possible sketch the proofs or provide references to those results that are left to the reader. Avoid the use of foreign phrases. Some authors from either sheer exuberance or a desire to show off will liberally sprinkle their work with Latin or French phrases. These phrases do not add anything to most writing, and will alienate many readers. If you are writing in English, then write in English. (Of course the exception to this is the use of foreign phrases such as "etc." or "déjà vu" which are used so commonly that they are now standard English.)

For many of the same reasons one should avoid obscure allusions when writing mathematics. Do not assume that your readers are familiar with Shakespeare's plays or the writings of Herodotus. Any allusions to such are are likely to be brushed off with irritation. Work on the assumption that your readers are narrow specialists and do not assume that they know anything beyond the prerequisite mathematics. Also be aware that many of your readers will be from foreign countries, and that references to Homer Simpson will annoy your Japanese readers just as much as references to the Shogun Tsunayoshi will annoy you.

Finally, be very careful about inserting humor into your writing. There have been very few mathematics books, and even fewer papers, that have gotten away with jokes. At best your attempt at humor falls flat, at worst it appears flippant or offends someone. This is more than just a matter of decorum. Keep in mind that your colleagues and others you respect will be reading your work. Also be aware that once in print your statements will become essentially permanent. When you look back at your work in five or ten years, what seemed funny at the time you wrote it may now make you feel embarrassed. Donald Knuth tells a story of how he once submitted a paper to the MONTHLY entitled *The Toilet Paper Problem*. The joke was continued throughout the paper with section names such as "An absorbing barrier", "A process of elimination", and "Residues". In the letter to the editor Knuth even joked that many of the MONTHLY's readers probably keep their copies in the bathroom anyway. Paul Halmos, the editor at the time, replied a little gravely that "jokes are dangerous in our journal" and asked Knuth to think twice about the scatological references. Knuth did agree to change the section names and remove the attempts at humor, but decided to leave the title intact since he had already given two talks announcing his results under this title. "Your toilet paper is accepted," replied Halmos. However, Knuth admits that to this day he still has occasional doubts when he catches sight of the title amid his list of publications.

6.7 Writing resources

Writing mathematics well requires many of the same skills that are necessary for excellence in other types of writing. At the same time, there are specific issues that are unique to mathematical writing. Because of the importance of quality writing in the mathematical profession, the beginning mathematician would do well to consult numerous sources in order to learn how to be a more effective writer.

The following are just a few references to books that deal with the mechanics, techniques, and instruments of writing.

The book

W. Strunk and E. B. White, *The Elements of Style*, 4$^{\text{th}}$ Ed., Macmillan, New York, 1979.

is the classic manual of style and I cannot recommend it enough. It is short enough to be read in a single sitting and the insights it contains are priceless. *The Elements of Style* has a reputation among authors as an indispensable tool for anyone who wants to write in any capacity. It distills the essence of writing concise, direct, and logical prose through principles that lead one to write clearly and succinctly.

A book which addresses the specifics of mathematical writing is

S. Krantz, *A Primer of Mathematical Writing*, American Mathematical Society, Providence, Rhode Island, 1997.

and I highly recommend it. It discusses the writing of many items (e.g., papers, theses, books, letters of recommendation, expository articles, email) and also gives a great deal of advice on functioning in the mathematical profession.

Some other references which are also useful (though in my opinion not as good as Krantz's book) are

L. Gillman, *Writing Mathematics Well: A Manual for Authors*, The Mathematical Association of America, Washington, D.C., 1987.

N. J. Higham, *Handbook of Writing for the Mathematical Sciences*, SIAM, Philadelphia, Pennsylvania, 1993.

American Mathematical Society, *A Manual for Authors of Mathematical Papers*, 8$^{\text{th}}$ Ed., pamphlet, 20 pp, American Mathematical Society, 1984.

Also when writing papers (mathematical or otherwise) it is essential that you have a good dictionary and a good thesaurus. You may also find it useful to have a dictionary of English usage. The standard reference is the following.

> H. W. Fowler, *A Dictionary of Modern English Usage*, 2nd Ed., Revised and Edited by Sir Ernest Gowers, Oxford University Press, 1965.

Chapter 7

Emotional factors in the process of research

"The study of mathematics is apt to commence in disappointment."

— *Alfred North Whitehead*

No student gets through the thesis writing process without a significant amount of stress. There is always too much to do and not enough time to do it. Sometimes it is not even clear what should be done. Students doubt their creativity, talent, and motivation. This is unavoidable.

You should be careful not to compare yourself to senior researchers who have many more years of work and publications. Also, be aware that it is common to find your estimation of your own work oscillating from "greatest story ever told" to "vacuous, redundant, and incoherent." This is normal. Keep correcting it with feedback from other people.

Recognition, in the form of positive feedback from your advisor or having a paper accepted for publication, can help with insecurity about progress. Talking to as many grad students, professors, and researchers as you can about your ideas is another way of receiving feedback on your work. Such people often contribute useful ideas and some of them are bound to have positive comments which will make you feel good. Since standards of progress are so tricky, it is easy to go down blind alleys if you are not in constant communication with other researchers. This is especially true when your work is not going well, ironically the time when you feel least like talking about it. It is especially important to get feedback and suggestions during these times.

It is easy not to see the progress you have made. You may think "If I can do it, it's trivial. My ideas are all obvious." They may be obvious to you in retrospect, but probably they are not obvious to anyone else. Explaining your work to others will help you keep in mind just how hard it is to understand what now seems trivial to you. You can do this through conversations with other graduate students, by talking with people at conferences, or by writing up and distributing preprints of your work.

> *"I cannot judge my work while I am doing it. I have to do as painters do, stand back and view it from a distance, but not too great a distance. How great? Guess."*
>
> — *Blaise Pascal*

A recent survey of a group of Nobel Laureates in science addressed the issue of self-doubt: The scientists were asked if it had it been clear all along that their work was important. The unanimous response (out of approximately 50 people) was that they were constantly doubting the value and correctness of their work. They all went through periods of feeling that what they were doing was irrelevant, obvious, or wrong. Since critical evaluation is part of any scientific progress, some amount of uncertainty over the value of the work is an inevitable and important part of the process.

> *"Mathematics is a cruel profession. Solving a mathematical problem is for most mathematicians an arduous and lengthy process which may take years, even a lifetime. The final conquest of the truth comes, if ever, inevitably tinged with disillusion, soured by the realization of the ultimate irrelevance of all intellectual endeavor."*
>
> — *Gian-Carlo Rota*

There are at least two emotional reasons people tolerate the pain of research. One is a drive, a passion for the problems. You do the work because you could not live any other way. Much of the best research is done that way. It has severe "burnout" potential, though. Another reason is that good research is fun. It is frustrating much of the time, but if a problem is interesting to you, then you can approach it as play and enjoy working on it. These two emotional states are not incompatible. However, it is wise to seek a balance if you aspire to sustain your work over a lifetime.

A month or two after you have completed a project such as a paper or thesis, it is common to think that your work looks utterly worthless. This backlash effect is the result of being bored and burned-out on the problem, and it is often caused by being able to see in retrospect that it could have been done better. Do not take this feeling seriously. You most likely will find that when you look back at it a year or two later, you will think "Hey, that's pretty clever. Nice piece of work!"

7.1 All work and no play . . .

Finding a balance between work, play, and other activities is not easy. Different people will give you very different advice. Some people say you should be spending all of your waking hours working on your thesis. Others (myself included) think that this is unrealistic and unhealthy, and that it is important for your mental and physical health to have other active interests.

One of the keys to balancing your life is to develop a schedule that is more or less consistent. You may decide that you will work mostly during the days, and that evenings are for your hobbies. Or you might decide that afternoons are for socializing and exercising, and work late at night.

Many graduate students hit the doldrums around the end of the third year, when they are trying to focus on a thesis topic and begin working on a problem. Sometimes this process can take quite a while. Try to find useful, enjoyable activities that can take your mind off of the thesis. Go to the movies in a film series, play a sport, read a novel, or knit. Also, if you schedule regular activities you will probably find it easier to avoid drifting aimlessly from day to day.

7.2 Burnout

7.2.1 Diagnosis of burnout

According to New York psychologist Herbert J. Freudenberger Ph.D., who coined the term, burnout is a state of fatigue or frustration brought about by a devotion to a cause, a way of life, or a relationship that failed to produce the expected reward. Burnout is a problem born of good intentions, because it happens when people deplete their energy by trying to reach unrealistic goals. The result is often that they lose touch with themselves and others.

The onset of burnout is slow. The early symptoms include a feeling of emotional and physical exhaustion; a sense of alienation, cynicism, impatience, negativism and feelings of detachment to the point that the individual begins to resent the work they are involved in and the people who are a part of that work. In extreme cases, the individual who once cared very deeply about a project or a group will be driven to the point that they no longer care at all.

The irony of burnout is that it happens to the same person who previously was brimming over with energy and new ideas when faced with new tasks or new situations. This type of person generally has high expectations of what can be accomplished. As time goes by and the goals are not achieved, the enthusiasm dies and listlessness sets in. Instead of lowering standards or accepting reality, frustration is bottled up and the individual tries even harder. The result is burnout.

Three factors are associated with burnout:

- role conflict: A person with conflicting responsibilities is at high risk of feeling as though they are pulled in many directions. They may try to do everything equally well without setting priorities. The result is often the feelings of fatigue or exhaustion associated with burnout.

- role ambiguity: The individual does not know what is expected of them. They know that they are expected to be a good career person but are not quite sure how to accomplish this because they have no role model or guidelines to follow. The result is that one feels as though they are never accomplishing anything worthwhile.

- role overload: The individual cannot say no and accepts more responsibility than they can handle.

7.2.2 Remedies for burnout

When you recognize burnout, spend some time reflecting. Try to remember when it was that you began feeling so tired and unable to relax. Were you always under such pressure to succeed? When did this one area of your life become disproportionately important? At what point did you lose your sense of humor and the personal side of your relationships with friends and coworkers? Have you identified so closely with your responsibilities that you have come to believe that if this project falls apart you have failed? The answers to these questions will help you reestablish your values and priorities.

The next step is to make some changes in your life. When your work begins to lose its appeal, it is time to change your duties or to take a break. The following is a list of additional remedies for burnout:

- Establish some long and short term goals that are realistic.

- If you have been neglecting your health, change your eating habits and begin to exercise more.

- Set aside some time each day to relax. You may also wish to try some type of relaxation exercises such as mediation or tai chi.

- Renew your friendships with other people. Talk to them about your feelings. Do not keep your frustrations and anger bottled up.

- Analyze how you spend your time. Try to incorporate some time management techniques into your life.

- Learn to say no when you are asked to do more than you can handle.

- Learn to delegate responsibility to others.

- Learn to recognize when you are driving yourself too hard and realize that it is counterproductive.

- Find the sense of humor you may have lost. Learn to laugh at yourself.
- Most of all, reevaluate your goals and determine what you want out of life.

> *"Being a graduate student is like being all of the Seven Dwarves. In the beginning you are Dopey and Bashful. In the middle, you are usually Sneezy, Sleepy, and Grumpy. But at the end they call you Doc, and then you are Happy."*
>
> — *unknown*

Appendix A

Myths about mathematics and graduate school

Myth: Graduate school is a 9 to 5, Monday to Friday job.

Reality: Success in graduate school requires a great deal of time and effort. Consequently, you will be required to complete tasks which cannot be accomplished within normal working hours. Grading for classes will often need to be done at night, conferences are frequently held on weekends, and preparing for Monday classes or seminars must often be done over the weekend. In addition, successful mathematicians do not stop thinking about their problems at 5PM every day. In order to obtain results you will often have to work for long periods of time and think about your problem every chance you have. This is not to say that you cannot take breaks. But if you have a good idea right before you plan to go home, it might be worth taking an extra hour or two to explore this idea while you are feeling inspired.

Myth: You can only work on math a few hours a day.

Reality: This myth is closely related to the previous one. Often individuals will claim that they can only work on a problem for a brief period of time, and that any additional effort only serves to make them burnt out. In order to be successful, you will need to learn to work on mathematics for extended periods of time. See *The Necessity of Structure* section in Chapter 2 for more regarding this topic.

Myth: You can write up your thesis and all your results in a month or two.

Reality: As mentioned in Chapter 6, writing is a long and arduous process. Writing up your thesis is a Herculean task that requires numerous months of work. If you intend to graduate in the spring, then you should begin writing your thesis the previous fall. Remember that your final year of graduate school will be filled with many additional tasks such as applying for jobs, going on interviews, and finishing any final requirements for graduation (e.g., language exams). It is not unreasonable to allow yourself a year to write up your results.

In addition, you will probably want to begin breaking your thesis into papers that you can submit for publication. Some people wait until after they graduate to do this. However, it is to your advantage to get started on it as soon as possible. Not only will the material be fresher in your mind, but having preprints may help when you are looking for a job.

Myth: Mathematicians work in complete isolation.

Reality: While some mathematicians do a great deal of independent work, no one works in complete isolation. Any individual mathematician is part of a larger research community and it is necessary that they realize this. The work of others will influence your work as well as how others view it. If you do not adopt the standard notation used by others working in your area or if you decide to use peculiar terminology without explaining it, then others may not take the time to read or your results.

Furthermore, collaboration is extremely important in mathematics. Most theorems are not proven by an individual sitting in a room with a pad of paper and a pencil, but rather arise from lively discussions, participation in seminars, and readings of the literature. In addition many papers are authored jointly. Joint work has several advantages. Not only do multiple authors provide additional sources for ideas and comments, but also tedious work, such as writing up results or proofreading, can be divided among the authors.

Myth: Mathematics is a realm of complete objectivity. Mathematics is nonhuman.

Reality: Mathematics, like any human endeavor, is filled with all the emotions, virtues, and prejudices that humanity possesses. As a result much of mathematics is subjective. Often the tastes, interests, and experiences of human mathematicians determine which questions are considered intriguing, which proofs are seen as beautiful, and which results are deemed important.

Myth: Mathematicians do their best work in their youth.

> *"The mathematical life of a mathematician is short. Work rarely improves after the age of twenty-five or thirty. If little has been accomplished by then, little will ever be accomplished."*
>
> — *Alfred Adler*

> *"Young men should prove theorems, old men should write books."*
>
> — *G.H. Hardy*

Reality: Age has little to do with mathematical ability. Creativity and originality, the hallmarks of good mathematical research, are often unaffected by age. In fact, the experience and knowledge that are obtained with age are often valuable assets in research. In addition, it is often the case among graduate students that the older (or nontraditional) students are more dedicated to mathematics than those who enter graduate school within a year or two of college. They are typically more established in their lives, and their decision to pursue mathematics is done with greater deliberation.

Myth: The most important thing that a mathematician does is research.

> *"There is no scorn more profound, or on the whole more justifiable, than that of the men who make for the men who explain. Exposition, criticism, appreciation, is work for second-rate minds."*
>
> — *G.H. Hardy*

Reality: We can all probably recall a truly gifted teacher in our past who inspired us to be a mathematician. It is likely that this person was also influential in the mathematical development of many others. Clearly the impact that this teacher has on the mathematical community is important. Think of some of the best researchers that you know. It is likely that they were inspired and influenced by teachers such as yours.

In addition, good survey articles in any subject are often as useful as research. They serve to organize and summarize current work for the specialist, and to provide an introduction to the topics for the nonspecialist. Littlewood has said that "A good mathematical joke is better, and better mathematics, than a dozen mediocre papers." I think the same can be said of a good expository article.

Myth: Mathematics is deductive.

Reality: Mathematics is a human activity. Consequently its creation does not proceed in a linear and predetermined way. When proving a theorem, one does not sit down and prove the necessary lemmas followed by the theorem and a corollary or two. Instead there are false starts, backtracking, and wild guesses. Often one ends up proving something entirely different than what was initially intended.

> *"Mathematics —this may surprise or shock some— is never deductive in its creation. The mathematician at work makes vague guesses, visualizes broad generalizations, and jumps to unwarranted conclusions. He arranges and rearranges his ideas, and becomes convinced of their truth long before he can write down a logical proof ... the deductive stage, writing the results down, and writing its rigorous proof are relatively trivial once the real insight arrives; it is more the draftsman's work not the architect's."*
>
> — *Paul Halmos*

> *"We have a habit in writing articles published in scientific journals to make the work as finished as possible, to cover up all the tracks, to not worry about the blind alleys or describe how you had the wrong idea first, and so on. So there isn't any place to publish, in a dignified manner, what you actually did in order to get to do the work."*
>
> — *Richard Feynman*

Appendix B

Writing tips

Writing is a significant and essential part of being a mathematician, and anyone who enters the profession will find their time occupied with the writing of mathematics papers, grant proposals, letters of recommendation, referee reports, and a variety of other items. It is often in graduate school that one first does substantial mathematical writing in the form of papers, a thesis, a research statement for job applications, and possibly also proposals for fellowships or grants.

The following is a list of writing suggestions that I have compiled after reading many graduate students' first attempts at mathematical writing. Writing mathematics well is a skill that takes time and practice to learn, and the following list is meant to provide the beginner with aspects of style to consider, conventions to be aware of, and common pitfalls to avoid. While much of the following may seem like common sense, keep in mind that it is surprisingly easy, even for experienced writers, to forget the following guidelines when immersed in writing mathematics. Therefore, reflect on the following list and make a conscious effort to incorporate its suggestions when you write.

B.1 Mechanics

1. Do not use common blackboard abbreviations. For example, write "if and only if" rather than "iff", and "without loss of generality" rather than "WLOG". This also applies to symbols such as $\forall$ and $\exists$. Unless one is writing a paper in mathematical logic, one should write out "for all" and "there exists".

2. Punctuate equations and mathematical symbols. Mathematical expressions are no different than the words they represent, and they should be punctuated accordingly. This applies even to displayed equations so that, for example, if a displayed equation is at the end of a sentence it should end with a period.

3. Do not use contractions in formal writing. Thus words such as "don't", "can't", "I'm", and "we've" should be written out.

4. Use the first person plural when writing mathematics papers. It is a convention in the mathematical community to use the first person plural, or "we", when writing papers. This choice has many advantages. It conveys the active and participatory nature of the project, making readers feel involved as they work through the paper. In addition, it is what most people are used to hearing in mathematics classes or talks, and therefore has a familiar cadence which is less likely to cause distraction. Furthermore, it avoids many of the problems encountered with other choices, such as the pretension of the first person singular "I", or the awkward sentences that arise with the third person singular "one".

5. Write all necessary hypotheses in statements of theorems. A person should be able to open your paper to any theorem, read it, and know what you are talking about without having to refer to earlier portions of the document. In fact, it is likely that this is how most people will use your papers. If at all possible, make the statements of your theorems completely self-contained so that the reader does not have to look throughout your paper to decipher notation or find definitions of special terminology.

6. Use Latin abbreviations correctly. The following table summarizes the meanings of some commonly used Latin abbreviations:

Abbreviation	**Latin term**	**English translation**
i.e.	id est	that is
e.g.	exempli gratia	for example
cf.	confer	compare
n.b.	nota bene	note well (or just note)
q.v.	quod vide	which see
viz.	videlicet	namely
et al.	et alii	and others

In particular, the abbreviations i.e. and e.g. are often mistakenly interchanged, and cf. is often misused to mean "see" when it actually means "compare". Also note that there is no period after "et" since it is not an abbreviation.

7. Do not start a sentence with a variable or symbol. Although it is perhaps technically correct, it is considered bad style to do so. Usually this can be avoided by simply rewording the sentence; e.g., rather than "n points are on the interior" one would write "The interior contains n points". (There are, of course, some exceptions to this rule. In particular, most people would consider it acceptable to start a sentence with a mathematical term that contains a symbol, especially if it is an uppercase symbol. For example, one should feel free to start a sentence with the word C^*-algebra or the word K-theory.)

B.2 Style

1. Use brevity in your writing. Write as simply and directly as possible. Avoid the use of ponderous or pretentious prose, and remove any unnecessary words or phrases. It is surprising how often one can take a piece of writing and improve it merely by removing portions. This does not mean that your final product must be short, or that you must leave out details. Instead, you should write so that every word, phrase, and sentence contributes to what you are trying to communicate. For example, there's no need to say: "and now we prove a lemma". Simply prove it. Likewise there is no reason to remark: "we have proven the claim" at the end of a proof, since the symbol $\square$ says precisely that already. Do not repeat yourself in your writing, and do not use superfluous phrases. When you proofread a paper for the first time, ask yourself after every sentence whether the reader would be any less informed you removed it. If the answer is "no", you should take it out. In short, you should follow the advice of William Strunk and "make every word tell".

> *"Vigorous writing is concise. A sentence should contain no unnecessary words, a paragraph no unnecessary sentences, for the same reason that a drawing should have no unnecessary lines and a machine no unnecessary parts. This requires not that the writer make all sentences short or avoid all detail and treat only subjects in outline, but that every word tell."*
>
> — *William Strunk*

Furthermore, keep in mind that writing concisely takes far more time and effort than writing at length — it is for this reason that Pascal, once at the end of a long letter, apologized for not having had the time to write a short one. However, writing in a concise and succinct manner is well worth the effort because it contributes to producing work that is clear, well organized, and direct.

2. Use language precisely and correctly. In mathematics more than any other subject one needs to be careful of word choice. Theorems must be stated carefully and unambiguously, arguments must be logical and clear, and exposition must convey what the writer intended. A mathematician is unlikely to use technical terms, such as "differentiable" or "Hausdorff", incorrectly. However when writing English prose it is common to be more sloppy as one tries to convey technical ideas with what is often imprecise language. Moreover, one should be careful of using words that may have unintended meanings. For instance, referring to an example of a ring as "simple" to mean that it is easy to understand could be misconstrued as meaning the ring has no ideals. Similarly, English words such as "complex", "trivial", and "natural" can be misinterpreted.

3. Organize your paper in an order that makes the exposition clear. In particular, this will not usually be the order of discovery. Often when proving theorems one will first obtain a collection of results, and then later prove a theorem or create a theory which encapsulates these results as special cases. When writing up these results, one may want to first prove a general theorem and then obtain the special cases as corollaries. On the other hand, it may be appropriate to begin with a few specific examples which identify the important concepts and motivate the more general work to follow. (In either case, however, one would not want to prove a specific theorem first, and then a more general theorem later, since this would result in unnecessary repetition.)

4. Write a good introduction. Most people who read a mathematics paper will only read the introduction and skim the theorems. Furthermore, when a reviewer reads a proposal for a grant or fellowship, it is the introduction which will have the most influence on the reviewer's opinion of your work. Consequently, you should put a great deal of time and effort into writing an effective introduction. Introduce the problem you are working on, motivate the solution, clearly state (or summarize) your results, and explain the significance of the solution. In addition, do your best to connect your work to the work of others, to other areas of mathematics, and to various applications. Remember that in an introduction you are often trying to "sell" your work and convince others of its importance. Also be aware that many people write the introduction to a paper after they have finished the body. This gives them the advantage of knowing exactly what will be done in the paper as they compose the introduction.

5. Write with the reader in mind. Identify an audience, and write with an awareness of that audience. As you write a mathematics paper remember that, unlike you, the reader has not been thinking intensely about the material for an extended period of time. Therefore, provide the reader with references, include useful comments, and give additional explication so that someone unfamiliar with the work can follow it. (I personally try to write papers in such a way that a graduate student in the area could read them.) In addition, when writing a proposal for a grant or fellowship, remember that the reader will be a mathematician that may not be in your area, or at the very least may be unfamiliar with the specifics of the subject you work on. Finally, be especially careful when writing for a nonmathematician. It is all too easy to assume your reader knows more mathematics than they actually do. If you're going to talk about continuous functions, keep in mind that most people don't even know what a function is, much less understand the concept of continuity, so you'll have to explain these ideas if you wish to use them. Also be careful of English language that is commonly used in mathematics, but is usually unfamiliar to the layman. For example, terms such as "if and only if", "contrapositive", or "nontrivial" are used so often in mathematics that one often forgets that a nonmathematician may not know what they mean.

6. Make sure your writing flows. Avoid writing a succession of loose sentences. Particularly when writing proofs, it is easy to become so engrossed in the mathematics that one forgets to pay attention to English style. The result is often a proof that reads " . . . and then . . . and then . . . and then . . .". Try to use a variety of words in proofs, such as "therefore", "consequently", "it follows that", "we see", "hence", or "thus".

7. Listen to criticism and learn from it. It is natural to become attached to your writing and even to be proud of it. Consequently, it is often difficult to receive criticism without taking it personally. Keep in mind that criticism and the comments of others are often your most valuable tools for improving your writing. Often you have invested so much in what you have written that you are incapable of putting yourself in the position of the uninitiated reader. Therefore, feedback from others can be an indispensable tool for making your writing clearer, identifying portions that are confusing, and anticipating your readers' reactions.

B.3 TeX suggestions

1. Use LaTeX rather than Plain TeX, AMS-TeX, or AMS-LaTeX. LaTeX has a more user-friendly interface as well as better documentation than any of these other formats. It is also the most commonly used format among mathematicians at the present time, and thus makes it easier to communicate with other mathematicians as well as submit papers to journals. Moreover, LaTeX has a large number of extension packages readily available. This means that you will often be able to achieve special effects by using one of the packages already in existence, instead of having to do it yourself. Furthermore, LaTeX creates "structured" files in which the various elements (title, authors, headings, etc.) are easily identified. This is not only useful for journals that wish to transfer manuscripts into new formats, but it also forces the author to organize his or her writing in a logical manner. Finally, LaTeX has better support for graphics than other TeX formats, making it easier to incorporate graphics consistently.

2. Make quotation marks correctly. Many beginners do not know how to properly make quotation marks in TeX. If you want to typeset "something in quotes" then you need to type

```
``something in quotes"
```

in your TEX source. That is, the opening quotes are made by typing the single opening quote

```
`
```

twice (this key is located in the upper left hand corner of the keyboard, to the left of the 1 key), and the closed quotes are made by typing the double quote

```
"
```

simultaneously pressing shift and the key next to the return key. If instead you make the mistake of typing

```
"something in quotes"
```

in your TEX source, then you will get ”something in quotes” in the typeset document. This may seem like a small mistake, but to anyone who is familiar with TEX it is an egregious error. If you frequently make this mistake you will annoy many of your readers as well as look like a complete novice.

3. Use macros, but use them judiciously. Macros are user-defined commands that are placed in the preamble of your document (in between the `\usepackage` lines and the `\begin{document}` line). Macros are useful for two main reasons. First, they can save you a lot of time by allowing you to create shortcuts. For example, if you make the symbol $D^{[2]} \times D^{[3]}$ frequently, you could define the macro

```
\newcommand{\DD}{D^{[2]} \times D^{[3]}}
```

which then allows you to type `\DD` to obtain the symbol. Creating such shortcuts can be a great time saver, but be careful not to overdo it or your document will be difficult for others to read. Another use of macros that many beginners are unaware of is that you can use them to define new operators. For example, if you want to create an operator such as Ext, then in the preamble you type

```
\newcommand{\ext}{\operatorname{Ext}}
```

and then in your document commands such as `$\ext A$` or `$\ext C(X)$` will result in the symbols $\operatorname{Ext} A$ and $\operatorname{Ext} C(X)$, respectively. In particular, note that this causes the operator to be written in Roman text as well as creates the proper amount of space between the operator and the argument. It is usually convenient to use a macro whenever you define a new operator.

4. Use the proper size of dash. There are four sizes of dash in TEX: the *dash*, the *en dash*, the *em dash*, and the *minus sign*. The dash (or hyphen) is used to hyphenate words, and is produced from a single - in the TEX source. For example,

```
non-negative
```

appears as non-negative when typeset. The en dash is slightly longer and used for page numbers. It is produced by two - in the TEX source. For example,

```
pages 32--45
```

appears as pages 32–45 when typeset. You should use the en dash when listing page numbers in your bibliographies. The em-dash is longer yet, and is used as a punctuation mark. It is produced by three - in the TEX source. For example,

```
This an em dash --- a type of punctuation.
```

will produce: This is an em dash — a type of punctuation. The final type of dash is the minus sign, which is produced by a single - appearing in math mode. For example,

```
$5 - 3 = 2$
```

will produce $5 - 3 = 2$. Note that the minus sign is slightly longer than a dash. Use the proper dashes in various situations. In particular, make sure that when you list page numbers in your bibliography you use the en dash.

5. Always spellcheck your TEX documents. There are various software packages available for spellchecking documents in the various implementations of TEX.

- Macintosh
  - Excalibur
    available for free download at

    http://excalibur.sourceforge.net/
- PC
  - jspell
    available at ftp://ftp.tex.ac.uk/pub/archive/support/jspell/
  - Microspell from Trigram Systems
  - TEXSpell
- Unix
  - ispell

6. If possible, type out the LaTeX source rather than using a Front End such as Scientific Word or Scientific Workplace. A Front End is a piece of software that incorporates a LaTeX preprocessor to typeset the document as you type. Often commands can easily be accessed through menus, the source document is not visible, and the overall experience is closer to WYSIWYG than with more basic LaTeX implementations. While these Front Ends are usually easier for the novice to learn, they can become a crutch if it is all that you are familiar with. Knowing how to manipulate the LaTeX source allows you greater flexibility as well as a lot more control over your documents. Furthermore, if at some point you need to use a new program for TeXing (for example, if you are working with colleagues or change institutions), it is fairly easy to switch from typing out the LaTeX source to using a Front End, but a bit more difficult to go the other way.

TeX Resources

Books on LaTeX:

G. Grätzer, *First Steps in LaTeX*, Birkhäuser, Boston, Springer-Verlag, New York, 1999.

G. Grätzer, *Math into LaTeX*, 4^{th} Ed., Birkhäuser, Boston, Springer-Verlag, New York, 2007.

There are three main sources on the web for TeX information:

- The TeX Users Group (TUG) is an organization that supports and promotes the use of TeX. The TUG web site http://www.tug.org contains some useful information as well as numerous links and answers to frequently asked questions. TUG publishes a quarterly journal (the TUGboat) and organizes an annual international conference.

- The Comprehensive TeX Archive Network (CTAN) is the preeminent collection of TeX-related material on the Internet. CTAN is located at the site http://www.ctan.org and contains links to many of the files and packages for TeX that are available for free download. It also contains freeware and shareware implementations of TeX such as emTeX, MiKTeX, and teTeX.

- The AMS also provides excellent technical advice for TeX users at the site http://www.ams.org/tex .

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