Peter Cameron's Blog

Counting the things that need to be counted

Posts Tagged 'Paul Halmos'

How to write mathematics

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The title of this post is that of an issue of *l'Enseignement Mathématique* from 1970, by Norman E. Steenrod, Paul R. Halmos, Menahem M. Schiffer and Jean A. Dieudonné, and in particular the article of nearly 50 pages by Halmos which recently fell into my hands. It is a classic which, somehow, I never managed to read before now. But it is always good to go back to the classics occasionally. Halmos has some good ideas to impart, so I will take the liberty of summarising what he says, and direct you to the article for the full monty. [I have added occasional comments of my own.]

Halmos realises the thanklessness of his task; he quotes contradictory opinions of three people who read the first draft: "Too long", "Too short", "Too many examples", "Not enough examples", "Break long proofs up into lemmas", "Don't break long proofs up into lemmas", and so on. He says that the article might better be entitled "How I write mathematics"; but see the last section.

The paper consists of a preface and twenty sections.

1. *There is no recipe, and what it is*: By the time you read this, you will have written several pieces of mathematics, and will be convinced that you know how to write, and have nothing to learn; but there are some basic principles of mathematical writing.
2. *Say something*: that is, don't say nothing, and don't say lots of things.
3. *Speak to someone*: Be clear about what group of people you are writing for; further, think of a particular person you know, and address that person.
4. *Organize first*: after you have had the joyous freedom of jotting down random ideas, undergo the discipline of producing a diagram of what you are going to write.
5. *Think about the alphabet*: for example, don't use Σ for an index set if you are going to sum over it; choose between $ax+by$ and $a_1x_1+a_2x_2$; distinguish between $\in$ and ε; and so on. He says,

   > A mathematician's nightmare is a sequence n_ε that tends to 0 as ε becomes infinite.

   [I'll add one of my favourites: the quadratic equation $xa^2+ya+z = 0$.]

6. *Write in spirals*: Write your chapters in the order 1, 2, 1, 2, 3, 1, 2, 3, 4, etc. Spill your heart in the first writing; you can clean it up later. Give each section a title, even if you are not going to use it; if you can't think of a

title, maybe you don't know what the section is about.

7. *Organise always*, in a spiral way as with the writing. If Section 1 is on vector spaces and Section 2 on linear dependence, check that you have included in Section 1 the examples you will need in Section 2. Mathematical writing can have subplots and clues, like detective stories. For example, metric spaces could be a subplot in a book on general topology.
8. *Write good English* (or French, Japanese, or whatever). Don't use either casual, lazy style, or pedantic, flowery style. The language must be both correct and unobtrusive.
9. *Honesty is the best policy*. Again a plea for unobtrusiveness of style: your job is to help understanding, not to show off. Be especially careful with "obvious" and its cognates. When you read your manuscript six months later, or explain it to someone, is it still obvious?
10. *Down with the irrelevant and the trivial*. Deal with trivial cases, don't hide them; but don't belabour them. A statement of a theorem should be just that, self-contained, no chit-chat, no superfluous hypotheses (and of course no missing hypotheses).
11. *Do and do not repeat*. If Theorem 2 is almost identical to Theorem 1, use identical words as far as possible, and draw attention to the difference with a drum roll. On the other hand, repeating exactly the same thing in the same words does not help understanding; if you need to say it again, say it in a different way. Finally, if the proof of Theorem 2 is almost identical to that of Theorem 1, that may be a sign that you don't really understand what is going on.
12. *The editorial "we" is not all bad*, but remember that it means "the writer and the reader"; it is not the royal "we".
13. *Use words correctly*. The word "any" is dangerous since it can be either a universal or an existential quantifier depending on context ("Have you any wool?", "Any number can play"); it is better to use "each" or "every", or recast the sentence. Other difficulties occur with "where", "equivalent", and "if … then if … then".
14. *Use technical terms correctly*. Don't say "The function z^2+1 is even"; distinguish between "set" and "sequence", and between "contains" and "includes". Above all, be consistent. Some special advice about words: "(1) Avoid technical terms, and especially the creation of new ones, whenever possible. (2) Think hard about the new ones you must create; consult Roget; and make them as appropriate as possible. (3) Use the old ones correctly and consistently, but with a minimum of obtrusive pedantry."
15. *Resist symbols*. For example: in "Every continuous function f is bounded", f is unnecessary; if you have proved an equation labelled (*) for a function f, don't say "g also satisfies (*)". Be careful about arguments with long chains of lines beginning with = signs: how will your reader follow them?
16. *Use symbols correctly*. Sometimes you need $\in$, sometimes "in", maybe both in the same sentence, as "For x in A, we have $x \in B$." Avoid having two formulae separated only by punctuation; this is a particular danger if a sentence begins with a formula. [I think you should avoid starting a sentence with a formula.]
17. *All communication is exposition*. The rules for writing a book apply, with minor modification, to writing a research paper, or to preparing a lecture.
18. *Defend your style* against editors and proof-readers. Halmos quotes the example of the sentence

    > p or q holds according as x is negative or positive

    where the ending of the sentence was changed to "positive or negative" by the copy-editor because "it sounded better".

19. *Stop* when you come to the end of what you need to say (even if there is lots more you could say!).
20. *The last word*: Do as I say, not as I do.

My students will be aware that I don't agree with everything that Halmos says. But I believe I have earned the right to disagree by having written a lot and thought about the process. I agree with almost everything, and I think it is good for even the most experienced mathematical writer to be confronted with articles like this from time to time. So go read it!