MATHEMATICAL WRITING: A BRIEF GUIDE

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"Wordsmithing is a much greater percentage of what I am supposed to be doing in life than I would have ever thought."

— Donald E. Knuth

Writing is a significant and essential part of being a mathematician, and anyone who enters the profession will find their time occupied with the writing of mathematics papers, grant proposals, letters of recommendation, referee reports, and a variety of other items. It is often in graduate school that one first does substantial mathematical writing in the form of papers, a thesis, a research statement for job applications, and possibly also proposals for fellowships or grants.

The following is a list of writing suggestions that I have compiled after reading many graduate students' first attempts at mathematical writing. Writing mathematics well is a skill that takes time and practice to learn, and the following list is meant to provide the beginner with aspects of style to consider, conventions to be aware of, and common pitfalls to avoid. While much of the following may seem like common sense, keep in mind that it is surprisingly easy, even for experienced writers, to forget the following guidelines when immersed in writing mathematics. Therefore, reflect on the following list and make a conscious effort to incorporate its suggestions when you write.

Mechanics

1. Do not use common blackboard abbreviations. For example, write "if and only if" rather than "iff", and "without loss of generality" rather than "WLOG". This also applies to symbols such as $\forall$ and $\exists$. Unless one is writing a paper in mathematical logic, one should write out "for all" and "there exists".

2. Punctuate equations and mathematical symbols. Mathematical expressions are no different than the words they represent, and they should be punctuated accordingly. This applies even to displayed equations so that, for example, if a displayed equation is at the end of a sentence it should end with a period.

3. Do not use contractions in formal writing. Thus words such as "don't", "can't", "I'm", and "we've" should be written out.

4. Use the first person plural when writing mathematics papers. It is a convention in the mathematical community to use the first person plural, or "we", when writing papers. This choice has many advantages. It conveys the

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active and participatory nature of the project, making readers feel involved as they work through the paper. In addition, it is what most people are used to hearing in mathematics classes or talks, and therefore has a familiar cadence which is less likely to cause distraction. Furthermore, it avoids many of the problems encountered with other choices, such as the pretension of the first person singular "I", or the awkward sentences that arise with the third person singular "one".

5. Write all necessary hypotheses in statements of theorems. A person should be able to open your paper to any theorem, read it, and know what you are talking about without having to refer to earlier portions of the document. In fact, it is likely that this is how most people will use your papers. If at all possible, make the statements of your theorems completely self-contained so that the reader does not have to look throughout your paper to decipher notation or find definitions of special terminology.

6. Use Latin abbreviations correctly. The following table summarizes the meanings of some commonly used Latin abbreviations:

Abbreviation	**Latin term**	**English translation**
i.e.	id est	that is
e.g.	exempli gratia	for example
cf.	confer	compare
n.b.	nota bene	note well (or just note)
q.v.	quod vide	which see
viz.	videlicet	namely
et al.	et alii	and others

In particular, the abbreviations i.e. and e.g. are often mistakenly interchanged, and cf. is often misused to mean "see" when it actually means "compare". Also note that there is no period after "et" since it is not an abbreviation.

7. Do not start a sentence with a variable or symbol. Although it is perhaps technically correct, it is considered bad style to do so. Usually this can be avoided by simply rewording the sentence; e.g., rather than "n points are on the interior" one would write "The interior contains n points". (There are, of course, some exceptions to this rule. In particular, most people would consider it acceptable to start a sentence with a mathematical term that contains a symbol, especially if it is an uppercase symbol. For example, one should feel free to start a sentence with the word C^*-algebra or the word K-theory.)

Style

1. Use brevity in your writing. Write as simply and directly as possible. Avoid the use of ponderous or pretentious prose, and remove any unnecessary words or phrases. It is surprising how often one can take a piece of writing and improve it merely by removing portions. This doesn't mean that your final product must be short, or that you must leave out details. Instead, you should write so that every word, phrase, and sentence contributes to what you are trying to communicate.

For example, there's no need to say: "and now we prove a lemma". Simply prove it. Likewise there is no reason to remark: "we have proven the claim" at the end of a proof, since the symbol □ says precisely that already. Do not repeat yourself in your writing, and do not use superfluous phrases. When you proofread a paper for the first time, ask yourself after every sentence whether the reader would be any less informed you removed it. If the answer is "no", you should take it out. In short, you should follow the advice of William Strunk and "make every word tell".

> *"Vigorous writing is concise. A sentence should contain no unnecessary words, a paragraph no unnecessary sentences, for the same reason that a drawing should have no unnecessary lines and a machine no unnecessary parts. This requires not that the writer make all sentences short or avoid all detail and treat only subjects in outline, but that every word tell."*
>
> — *William Strunk*

Furthermore, keep in mind that writing concisely takes far more time and effort than writing at length — it is for this reason that Pascal, once at the end of a long letter, apologized for not having had the time to write a short one. However, writing in a concise and succinct manner is well worth the effort because it contributes to producing work that is clear, well organized, and direct.

2. Use language precisely and correctly. In mathematics more than any other subject one needs to be careful of word choice. Theorems must be stated carefully and unambiguously, arguments must be logical and clear, and exposition must convey what the writer intended. A mathematician is unlikely to use technical terms, such as "differentiable" or "Hausdorff", incorrectly. However when writing English prose it is common to be more sloppy as one tries to convey technical ideas with what is often imprecise language. Moreover, one should be careful of using words that may have unintended meanings. For instance, referring to an example of a ring as "simple" to mean that it is easy to understand could be misconstrued as meaning the ring has no ideals. Similarly, English words such as "complex", "trivial", and "natural" can be misinterpreted.

3. Organize your paper in an order that makes the exposition clear. In particular, this will not usually be the order of discovery. Often when proving theorems one will first obtain a collection of results, and then later prove a theorem or create a theory which encapsulates these results as special cases. When writing up these results, one may want to first prove a general theorem and then obtain the special cases as corollaries. On the other hand, it may be appropriate to begin with a few specific examples which identify the important concepts and motivate the more general work to follow. (In either case, however, one would not want to prove a specific theorem first, and then a more general theorem later, since this would result in unnecessary repetition.)

4. Write a good introduction. Most people who read a mathematics paper will only read the introduction and skim the theorems. Furthermore, when a reviewer reads a proposal for a grant or fellowship, it is the introduction which will have the most influence on the reviewer's opinion of your work. Consequently, you should put a great deal of time and effort into writing an effective introduction. Introduce the problem you are working on, motivate the solution, clearly state (or summarize)

your results, and explain the significance of the solution. In addition, do your best to connect your work to the work of others, to other areas of mathematics, and to various applications. Remember that in an introduction you are often trying to "sell" your work and convince others of its importance. Also be aware that many people write the introduction to a paper after they have finished the body. This gives them the advantage of knowing exactly what will be done in the paper as they compose the introduction.

5. Write with the reader in mind. Identify an audience, and write with an awareness of that audience. As you write a mathematics paper remember that, unlike you, the reader has not been thinking intensely about the material for an extended period of time. Therefore, provide the reader with references, include useful comments, and give additional explication so that someone unfamiliar with the work can follow it. (I personally try to write papers in such a way that a graduate student in the area could read them.) In addition, when writing a proposal for a grant or fellowship, remember that the reader will be a mathematician that may not be in your area, or at the very least may be unfamiliar with the specifics of the subject you work on. Finally, be especially careful when writing for a nonmathematician. It is all too easy to assume your reader knows more mathematics than they actually do. If you're going to talk about continuous functions, keep in mind that most people don't even know what a function is, much less understand the concept of continuity, so you'll have to explain these ideas if you wish to use them. Also be careful of English language that is commonly used in mathematics, but is usually unfamiliar to the layman. For example, terms such as "if and only if", "contrapositive", or "nontrivial" are used so often in mathematics that one often forgets that a nonmathematician may not know what they mean.

6. Make sure your writing flows. Avoid writing a succession of loose sentences. Particularly when writing proofs, it is easy to become so engrossed in the mathematics that one forgets to pay attention to English style. The result is often a proof that reads " . . . and then . . . and then . . . and then . . .". Try to use a variety of words in proofs, such as "therefore", "consequently", "it follows that", "we see", "hence", or "thus".

7. Listen to criticism and learn from it. It is natural to become attached to your writing and even to be proud of it. Consequently, it is often difficult to receive criticism without taking it personally. Keep in mind that criticism and the comments of others are often your most valuable tools for improving your writing. Often you have invested so much in what you have written that you are incapable of putting yourself in the position of the uninitiated reader. Therefore, feedback from others can be an indispensable tool for making your writing clearer, identifying portions that are confusing, and anticipating your readers' reactions.

TEX Suggestions

1. Use LATEX rather than Plain TEX, AMS-TEX, or AMS-LATEX. LATEX has a more user-friendly interface as well as better documentation than any of these other formats. It is also the most commonly used format among mathematicians at

the present time, and thus makes it easier to communicate with other mathematicians as well as submit papers to journals. Moreover, LaTeX has a large number of extension packages readily available. This means that you will often be able to achieve special effects by using one of the packages already in existence, instead of having to do it yourself. Furthermore, LaTeX creates "structured" files in which the various elements (title, authors, headings, etc.) are easily identified. This is not only useful for journals that wish to transfer manuscripts into new formats, but it also forces the author to organize his or her writing in a logical manner. Finally, LaTeX has better support for graphics than other TeX formats, making it easier to incorporate graphics consistently.

2. Make quotation marks correctly. Many beginners do not know how to properly make quotation marks in TeX. If you want to typeset "something in quotes" then you need to type

```
``something in quotes"
```

in your TeX source. That is, the opening quotes are made by typing the single opening quote

```
`
```

twice (this key is located in the upper left hand corner of the keyboard, to the left of the 1 key), and the closed quotes are made by typing the double quote

```
"
```

which is accomplished by simultaneously pressing shift and the key next to the return key. If instead you make the mistake of typing

```
"something in quotes"
```

in your TeX source, then you will get "something in quotes" in the typeset document. This may seem like a small mistake, but to anyone who is familiar with TeX it is an egregious error. If you frequently make this mistake you will annoy many of your readers as well as look like a complete novice.

3. Use macros, but use them judiciously. Macros are user-defined commands that are placed in the preamble of your document (in between the `\usepackage` lines and the `\begin{document}` line). Macros are useful for two main reasons. First, they can save you a lot of time by allowing you to create shortcuts. For example, if you make the symbol $D^{[2]} \times D^{[3]}$ frequently, you could define the macro

```
\newcommand{\DD}{D^{[2]} \times D^{[3]}}
```

which then allows you to type `\DD` to obtain the symbol. Creating such shortcuts can be a great time saver, but be careful not to overdo it or your document will be difficult for others to read. Another use of macros that many beginners are unaware of is that you can use them to define new operators. For example, if you want to create an operator such as Ext, then in the preamble you type

```
\newcommand{\ext}{\operatorname{Ext}}
```

and then in your document commands such as `$\ext A$` or `$\ext C(X)$` will result in the symbols $\operatorname{Ext} A$ and $\operatorname{Ext} C(X)$, respectively. In particular, note that this causes the operator to be written in Roman text as well as creates the proper amount of space between the operator and the argument. It is usually convenient to use a macro whenever you define a new operator.

4. Use the proper size of dash. There are four sizes of dash in TEX: the *dash*, the *en dash*, the *em dash*, and the *minus sign*. The dash (or hyphen) is used to hyphenate words, and is produced from a single - in the TEX source. For example,

```
non-negative
```

appears as non-negative when typeset. The en dash is slightly longer and used for page numbers. It is produced by two - in the TEX source. For example,

```
pages 32--45
```

appears as pages 32–45 when typeset. You should use the en dash when listing page numbers in your bibliographies. The em-dash is longer yet, and is used as a punctuation mark. It is produced by three - in the TEX source. For example,

```
This an em dash --- a type of punctuation.
```

will produce: This is an em dash — a type of punctuation. The final type of dash is the minus sign, which is produced by a single - appearing in math mode. For example,

```
$5 - 3 = 2$
```

will produce $5-3=2$. Note that the minus sign is slightly longer than a dash. Use the proper dashes in various situations. In particular, make sure that when you list page numbers in your bibliography you use the en dash.

5. Always spellcheck your TEX documents. There are various software packages available for spellchecking documents in the various implementations of TEX.

- Macintosh
  - Excalibur
    available for free download at
    http://excalibur.sourceforge.net/
- PC
  - jspell
    available at ftp://ftp.tex.ac.uk/pub/archive/support/jspell/
  - Microspell from Trigram Systems
  - TEXSpell
- Unix
  - ispell

6. If possible, type out the LATEX source rather than using a Front End such as Scientific Word or Scientific Workplace. A Front End is a piece of software that incorporates a LATEX preprocessor to typeset the document as you type. Often commands can easily be accessed through menus, the source document is not visible, and the overall experience is closer to WYSIWYG than with more basic LATEX implementations. While these Front Ends are usually easier for the novice to learn, they can become a crutch if it is all that you are familiar with. Knowing how to manipulate the LATEX source allows you greater flexibility as well as a lot more control over your documents. Furthermore, if at some point you need to use a new program for TEXing (for example, if you are working with colleagues or change institutions), it is fairly easy to switch from typing out the LATEX source to using a Front End, but a bit more difficult to go the other way.

Resources

On mathematical writing:

S. Krantz, *A Primer of Mathematical Writing*, American Mathematical Society, Providence, Rhode Island, 1997.

P. Halmos, *How to Write Mathematics*, American Mathematical Society, Providence, Rhode Island, 1973.

L. Gillman, *Writing Mathematics Well: A Manual for Authors*, The Mathematical Association of America, Washington, D.C., 1987.

N. J. Higham, *Handbook of Writing for the Mathematical Sciences*, SIAM, Philadelphia, Pennsylvania, 1993.

American Mathematical Society, *A Manual for Authors of Mathematical Papers*, 8th Ed., pamphlet, 20 pp, American Mathematical Society, 1984.

On general writing and English usage:

W. Strunk and E. B. White, *The Elements of Style*, 4th Ed., Macmillan, New York, 1979.

H. W. Fowler, *A Dictionary of Modern English Usage*, 2nd Ed., Revised and Edited by Sir Ernest Gowers, Oxford University Press, 1965.

LaTeX resources:

G. Grätzer, *First Steps in LaTeX*, Birkhäuser, Boston, Springer-Verlag, New York, 1999.

G. Grätzer, *Math into LaTeX*, 4th Ed., Birkhäuser, Boston, Springer-Verlag, New York, 2007.

There are three main sources on the web for TeX information:

- The TeX Users Group (TUG) is an organization that supports and promotes the use of TeX. The TUG web site http://www.tug.org contains some useful information as well as numerous links and answers to frequently asked questions. TUG publishes a quarterly journal (the TUGboat) and organizes an annual international conference.
- The Comprehensive TeX Archive Network (CTAN) is the preeminent collection of TeX-related material on the Internet. CTAN is located at the site http://www.ctan.org and contains links to many of the files and packages for TeX that are available for free download. It also contains freeware and shareware implementations of TeX such as emTeX, MiKTeX, and teTeX.
- The AMS also provides excellent technical advice for TeX users at the site http://www.ams.org/tex .