Letters to the Editor

Three Lessons I Wish I Had Never Been Taught by Gian-Carlo Rota

I have the greatest admiration for Gian-Carlo Rota's mathematics, energy, talent, wit, personality, charisma, enthusiasm, and the list could (and perhaps should) be continued. However, the purpose of this letter is to point out that by publishing his "Ten Lessons I Wish I Had Been Taught" in the January 1997 issue of the AMS *Notices* (cf. p. 22–25) without any appropriate footnote indicating that some of the "lessons" are meant to be just read and enjoyed and must not be taken seriously without parental supervision, you (and, indirectly, Gian-Carlo Rota) may have caused irreparable damage to a large segment of our young (and not so young) mathematicians who are naive beyond belief and who will accept any advice no matter how absurd and nonsensical it is from such an international authority and supernova as Gian-Carlo Rota.

More specifically, Gian-Carlo Rota advises to "Publish the Same Result Several Times" and then uses the example of F. Riesz to illustrate his point. Poor F. Riesz and poor advice. First, F. Riesz was a perfectionist beyond help (just like yours truly). Second, his mother tongue was Hungarian, whereas his working languages were Hungarian, German, and to some extent English; and he lived in an era when the international language of mathematics was transforming towards (broken) English. Especially the latter played a major role in why some of his results appeared more than once in print. Gian-Carlo Rota writes, "Riesz's example is well worth following today." If he means that it's worth following (F. or M.) Riesz's taste, talent, and depth, then I couldn't agree more. Let me put it this way: if you are a mathematician of (F. or M.) Riesz's caliber, please do whatever you want. Otherwise, please do not contribute indiscriminately to the information junkyard, and please publish only "final" definitive forms of your results, and spare us from being bombarded by ϵ-improvements and generalizations.

By the way, Gian-Carlo Rota writes, "I bought a copy of Frederick Riesz's *Collected Papers* as soon as the big, thick, heavy, oversize volume was published." I wonder if he realizes that he was grossly cheated, because it's really *two* big, thick, heavy, oversize volumes.

Then Gian-Carlo Rota advises "Do Not Worry about Your Mistakes" and brings up Hilbert and himself to make his point. Well, who the heck should worry about them if not the one who is responsible for them? This is nightmarish advice for readers and editors alike. In addition, it adds to the instability of our mathematical structure: just imagine theorem after theorem proved using erroneous results (sound familiar?). My advice is: forget it, and please *do* keep worrying about all your past, present, and future mistakes.

About the Cover

It is possible to deform a plane in three-space into a ruled surface containing one triple point and two pinch points using equations $(x, y^3 - cy,\ xy + y^5 - cy^3)$, as described in the work of David Mond and Washington Marar. This rendering was produced at the Geometry Center by Thomas Banchoff of Brown University and Davide Cervone of Union College. It was included in the show "Surfaces beyond the Third Dimension" at the Providence Art Club in March 1996. For more information about this image, see `http://www.geom.umn.edu/locate/tfb/art/`.

Then Gian-Carlo Rota tells the reader to "Give Lavish Acknowledgments". Funny but phony, and it may ruin one's credibility. My advice is: give fair acknowledgments, no more and no less. Never say "I would like to thank Professor X…." If "you would like", then please do so and say "I thank…" (if I recall correctly, this comes from Paul Halmos). Don't use the word "Professor"; it's superfluous.

Acknowledgment. I would like to sincerely thank Professor Gian-Carlo Rota for the remaining "Seven Lessons I Wish I Had Been Taught". I wish I had the words to describe how much his advice means to me. I also would like to thank the readers of my letter for their wisdom and infinite patience. If I may, I would also like to ask the readers to send me a complete set of their reprints so that I could lavishly refer to their invaluable mathematical contributions for years to come, whether or not they contradict my own results.

Paul Nevai
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Received December 12, 1996

P.S. When "Ten Lessons I Wish I Had Been Taught" was originally published in *Concerns of Young Mathematicians*, Vol. 4, Issue 25, August 21, 1996, it was preceded by "The views expressed here do not necessarily represent those of the administrative board or membership of the Young Mathematicians' Network" (cf. `http://www.math.usouthal.edu/brick/ymn/V4/vol4.25.html`).

P.P.S. Gian-Carlo Rota's recently published *Indiscrete Thoughts* (Birkhäuser, 1997) contains several chapters in the same spirit as the subject of this letter. Although I have not had a chance to read the entire book yet and therefore am in no position to praise or criticize it, I am sure that once I finish reading the book, I will heartily recommend it to all mathematicians, dead or alive.

Withdraw Endorsement of NCTM Standards

I would like to thank Professor Wu for his insightful article on the mathematics reform movement published in the December 1996 "Forum". Professor Wu has asked if the rank-and-file members of the AMS agree with the endorsement of the NCTM Standards by the AMS leadership. I do not. I respectfully urge the AMS leadership to withdraw its endorsement of the NCTM Standards.

The NCTM Standards lack balance and downgrade the importance of basic skills. They have spawned the disastrous California Mathematics Framework, which, for example, advocates access to calculators for all kindergarten and elementary school students. The NCTM Standards have paved the way for elementary school pedagogies such as MathLand, which fails to explicitly develop the standard multiplication algorithm for elementary school students, and high school pedagogies such as Interactive Mathematics Program (IMP), which delays presenting the quadratic formula until the twelfth grade. These are not even the worst of the so-called reform pedagogies.

When research mathematicians lend credibility to these tendencies, it has a bullying effect on teachers and parents who object to extremist pedagogies in the reform movement. School administrators can point out, and some already have pointed out, that the AMS, the leading organization for U.S. mathematicians, supports the NCTM Standards on which the reform pedagogies are based.

When research mathematicians further endorse reform versions of calculus, like the "Harvard Calculus", which radically deemphasize the use of high school algebra, this lends support for high schools to teach less algebra. And many are doing just that.

In my opinion, the AMS leadership has created barriers to criticism of the mathematics reform movement through its editorial decisions and its appointments to committees which can influence mathematics pedagogy. I urge greater openness on the part of the AMS leadership toward criticisms of the mathematics reform movement.

David Klein
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Received December 16, 1996

Mathematics Teaching at Illinois

In the editorial of the *Notices of the AMS*, Vol. 44, Number 1, signed by Steven G. Krantz, it is stated: "At the University of Illinois in Urbana the engineers have started teaching their own math courses…." I wish to point out that this statement has no basis in fact. It is either an error or a product of the imagination.

Philippe Tondeur
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Received December 19, 1996

Mathematics Reform at Second-Tier Institutions

Professors Wu ["Forum", December 1996] and Mac Lane ["Letters to the Editor", December 1996] express criticisms of the current wave of "reforms" in the teaching of mathematics. We feel that such criticisms are long overdue and wish to congratulate both authors for expressing their opinions.

Wu and Mac Lane may not have first-hand experience of the status of the "calculus reform" at second- (and third-) tier four-year institutions (as we do). At these institutions the "reforms" hold greater sway than at institutions where scholarship is more highly valued. At such institutions, in our experience, political activists from the administration, from the school of education, from those with a financial stake in the "reform movement", and from governmental "politically correct" funding organizations promote such reforms (and are sometimes joined by a contingent of resident activist mathematicians). In the process, all distinctions between "proof", "explanation", and "observation" are often lost. In such schools, mathematics programs are often extremely starved for resources, even when compared to other programs at the same schools. In such a situation, a little seed money from cynical or ignorant book publishers, calculator salesmen, and Washington bureaucrats can have an immense impact. Official-looking flyers from commercial interests too often im-

press administrators (who are frequently more versed in "cooperative learning" than mathematics). Occasional letters critical of the "reforms" from famous mathematicians (famous, that is, to other mathematicians) are printed in journals outside the reading circle of ordinary administrators.

In many such mathematics departments, Ph.D. mathematicians are outnumbered by those with degrees from the school of education or those who lack advanced degrees. One can easily guess the effect this situation has on the current employment opportunities for new Ph.D.s in our field. It is not uncommon to observe students in class circled in groups of four or five with their graphing calculators and four-colored, expensive, and faddish calculus books attempting, in a few hours, to empirically rediscover the great insights of Newton, Leibniz, and Gauss. Their "discoveries", expressed on poster boards or on group tests, are recorded frequently in "collective grades". These "educational" procedures differ in essential ways from the more successful methods used to teach mathematics in the past. Instead of developing deep insights and a love for the beauty of mathematics, our students make observations with their calculators and look for patterns (much as our colleagues in experimental psychology [do]). Theory makes such students positively uncomfortable.

The "reform" movement, in its religious fever and intolerance, declares that the teaching of mathematics must change to reflect modern technology, the demands of the employment market, and the increasingly fickle taste of our students (with their abysmal algebraic skills). Ironically, these untested "reforms" forced on second-tier colleges and public schools have coincided with students *less* enthusiastic about mathematics, *less* able to apply what they have learned (even to easy, artificial problems, much less to "real-world" problems), and, in general, *less* familiar and knowledgeable about mathematics. Lost in all the sloganeering is the fact that *real* mathematics, as taught by *real* mathematicians, is becoming a rarer and rarer experience at American second- and third-tier universities and colleges. Mathematics without precision, rigor, and proof, while not totally lacking in value, is simply not mathematics!

In endorsing the NCTM Standards, the AMS implied it was speaking for its members. This is not so for the undersigned (both members of the AMS). Our deep appreciation to Doctors Wu and Mac Lane for sounding the alarm on this threat to our profession and to our students.

Boris A. Kushner
Marc H. Melman
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(Received December 16, 1996)

A Cyclic Pattern in Ph.D. Awards

Does anyone have an explanation for the strong biennial cycle in mathematics Ph.D.s, shown in Table 5 (and the first graph accompanying that table) on p. 1499 of the December 1996 *Notices*? Starting in 1989 (for simplicity I will write "1989" for "88–'89", i.e., "Fall '88 and Spring '89 degrees", and likewise for other years), every odd-numbered year has shown more U.S. citizen doctoral recipients than the preceding year, while every even-numbered year has shown *fewer* than the preceding year. Looking at even- and odd-numbered years separately, we see two rather smooth growth curves, one above the other.

Is there some two-year cycle of funding? Could the way the information is gathered be changing in a two-year cycle? Do related disciplines show the same cycle?

Subtracting U.S. citizen recipients from total recipients, it appears that noncitizen recipients were affected by the same cycle from 1991 through 1995. (Perhaps the cycle was present over the whole period but masked by a stronger and less regular overall rise than for U.S. recipients.) Among U.S. recipients, male and female recipients both show the same alternation of increases and decreases for the full period mentioned. I have also checked my own department's Ph.D. lists, and they have shown the same cycle even longer—since 1985—except for irregular peaks in the even-numbered years 1988 and 1994.

The unemployment information in the same article does not show a two-year cycle. However, it has been suggested to me that the cycle in Ph.D.s may be driven by "hidden unemployment", which could be looked for in the future by adding to the questionnaire to new Ph.D.s an item asking whether the respondent attempted a job search the preceding year and delayed graduation partly due to an unsatisfactory outcome. If true, this would just be a start to explaining the mystery.

George Bergman
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Received December 18, 1996

Preparation of Future Teachers

Both H. Wu in "The Mathematician and the Mathematics Education Reform" (*Notices*, December 1996) and Hyman Bass in "Mathematicians as Educators" (*Notices*, January 1997) suggest that mathematicians should pay much more attention to pedagogy at all levels. I agree. I would like to recount some of the efforts in this direction of the mathematics department at UC Davis, whose experiences may be instructive.

Over twenty years ago we introduced an MAT (Master of Arts in Teaching) degree, designed to prepare mathematically strong teachers at the elementary and secondary levels. This is a two-year program during which the student takes special mathematics courses, such as the history of calculus, and has extensive practice teaching, closely monitored by peers, professors, and resident teachers. Unfortunately, we were too successful. I estimate that over 80 percent of our graduates went into teaching at community colleges either directly or after a brief stint at the secondary level. (Three reasons: better pay, shorter hours, no discipline problem.)

We also introduced a two-quarter undergraduate course to provide prospective elementary teachers with a strong mathematics background. On the first day students are asked to

write a mathematical autobiography. Almost all mention that they dislike or fear mathematics and trace that attitude back to an episode in an elementary mathematics class. No wonder they tend to avoid science and mathematics courses and put off our course till their senior year.

The main objective of the instructor in that course is to change students' attitudes toward mathematics. Very few instructors can cope with such a challenge. Years spent in graduate school mastering analysis, algebra, and topology unfit most of us for empathizing with students who dislike mathematics.

Many mathematicians become interested in the way mathematics is taught when their own children enter school. But that interest does not qualify them to teach such a class. Someone who is going to teach prospective teachers should visit many classrooms and see the different ways of organizing a class (lecture, small-group, etc.), even try to teach a class over a period of weeks.

However, someone who wants to do a good job in mathematics education will not have the time or energy to continue mathematics research. If the campus or department does not appreciate the effort, that instructor may become a second-class citizen. On the other hand, if a department views mathematics education as an application of mathematics (as it views fluid flow, for instance), then the instructor may expect appreciation and promotion.

Mathematicians are probably far more involved in precollege education than they realize. They may be surprised to learn how many of their students who earn a bachelor's degree in mathematics become teachers. When they do learn, they may ask, "Is the present curriculum, which is designed for other purposes, the best preparation for future teachers?" The answer will likely be no, for the same reason that a curriculum chosen to prepare future teachers may not be ideal for preparing students for research or industry.

Anyone interested in mathematics education should become familiar with two books published by NCTM, *Curriculum and Evaluation Standards for School Mathematics* (1989, 258 pp.) and *Professional Standards for Teaching Mathematics* (1991, 196 pp.), which describe quite specifically how mathematics should be taught. These volumes, both endorsed by the AMS and the MAA, also strongly influence what textbooks will be published. The slogan of *The Mathematics Teacher*, the main journal of the NCTM, shows the importance of the *Standards*. It says that "The mission of NCTM is…that every student is ensured an equitable *Standards*-based mathematics education. …" This is quite a contrast with its ecumenical slogan up to 1995: "*The Mathematics Teacher* is devoted to improving mathematics instruction."

Pages 132–143 of the second volume are of particular interest to mathematicians, for they describe the mathematics a teacher should know in order to teach the *Standards* way. This critical section begins:

> Knowledge of both the content and discourse of mathematics is an essential component of teachers' preparation for the profession. Teachers' comfort with, and confidence in, their own knowledge of mathematics affects both what they teach and how they teach it. Their conceptions of mathematics shape their choice of worthwhile mathematical tasks, the kinds of learning environments they create, and the discourse in their classrooms.

All of us would agree with that. The first stage in the reform movement should have been to improve the mathematical knowledge of present and prospective elementary teachers. Unfortunately, the cart of curriculum reform has been put before the horse of well-prepared teachers. In fact, not a single article on the subject of the mathematical preparation of teachers has appeared in *The Mathematics Teacher* since the second *Standards* volume was published.

Because the AMS and MAA presumably agree with those twelve most crucial pages, these organizations should persuade mathematics departments to implement the recommendations made there. If all teachers were mathematically well prepared, I for one would stop worrying about the age-old battle still raging between "back to basics" and "understanding".

On the other hand, if mathematics departments do nothing to improve school mathematics, they should stop complaining that incoming freshmen lack mathematical skills.

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Received December 24, 1996