Division. Long division can be performed by analogy using the following procedure. To divide CCCLXXVII by XV as shown in Example 4, write the divisor pattern for reference as before at the top, the dividend pattern in line (b), and the quotient in line (a). A partial divisor is formed by shifting the divisor pattern as far to the left as possible with respect to the dividend. The quotient character is written in the same column as the unit position of the shifted pattern, which determines whether it is Etruscan. The divisor is written beneath the dividend once if the quotient is Etruscan, or from one to four times if it is not (in line (c) it is used twice). The lines are then subtracted, and the process is repeated until the remainder is less than the quotient.

These examples demonstrate clearly the remarkable simplicity of multiplication and division with Roman numerals. Key features are:

—Although place value is not used in the numeral system, it is introduced in the counting table just as it is in the abacus. The combination of numerals with the table forms a place value system with a repeating multiplicative pattern of 5-2-5-2.

—The explicit zero is not needed in the numeral system or in the calculating table with its fixed columns.

—Most important, the algorithms *do not* require a knowledge of the multiplication tables. Sequential digit systems with place values require tables for calculating, but Roman numerals with their repeating pattern require only the two shift rules in the calculating table.

I wish to express my thanks to Jane I. Robertson for her generous help and suggestions during the preparation of this paper, and also to A. Cooperband for comments and help in developing the final forms of the algorithms.

References

1. Jane I. Robertson, How to do arithmetic, this MONTHLY, 86 (1979), 431–439.
2. L. Hogben, Mathematics for the Million, W. W. Norton, New York, 1967.

AN APPLICATION OF POINCARE'S RECURRENCE THEOREM TO ACADEMIC ADMINISTRATION

KENNETH R. MEYER
Department of Mathematical Sciences, University of Cincinnati, Cincinnati, OH 45221

The present trend in science is to apply classical mathematics to nontraditional areas. This note gives an application of a classical theorem of dynamical systems to a long neglected area of study, academic administration, and thus proves that scientific research and academic administration are not mutually disjoint. It is the author's hope that other administrations will apply their early training in scientific research to study the quagmire into which they have slipped and thus carry forth this work.

A recurrent orbit in a system is one that returns infinitely often arbitrarily close to its initial position. Poinçaré's recurrence theorem [1] states: *In a compact conservative system almost all orbits are recurrent*. Poincaré discovered this theorem in his investigations into the motion of celestial bodies, and until now it has not had applications to such terrestrial matters as academic administrative structures. However, we shall show that this theorem can easily be applied to explain an often observed phenomena.

LEMMA 1. *An academic administrative system is conservative.*

Proof. All decisions are made by applying the principle of least action and therefore the system is conservative by a classical theorem of Maupertuis [2].

LEMMA 2. *An academic administrative system is compact.*

Proof. The system is governed by a finite number of arbitrarily short-sighted deans and is compact by definition.

Lemmas 1 and 2 verify the hypothesis of Poincaré's recurrence theorem and therefore the conclusions hold for all academic administrations. An immediate consequence of this result is:

THEOREM 1. *Almost all administrators vacillate.*

Finally, since many conservative systems are reversible, an administrator will not only return infinitely often to the same position but must have been there infinitely often in the past.

This paper was presented at a meeting of the chairmen of the Ohio State University system in Columbus, Ohio, October 1978.

References

1. H. Poincare, Les Méthodes Nouvelles de la Mécanique Céleste, Gauthier-Villars, Paris, 1892.
2. E. T. Whittaker, A Treatise on the Analytic Dynamics of Particles and Rigid Bodies, Cambridge University Press, 1904.

SUM-PRESERVING REARRANGEMENTS OF INFINITE SERIES

PAUL SCHAEFER
Department of Mathematics, SUNY, College of Arts and Science at Geneseo, Geneseo, NY 14454

1. Introduction. Every student of advanced calculus knows that an absolutely convergent series of real numbers may be rearranged in an arbitrary fashion to obtain a new series which converges to the same sum as that of the original series. The student also finds that conditionally convergent series behave somewhat differently in this regard. Indeed, Riemann proved that such series can be rearranged to converge to any arbitrary real number, or even to diverge. Moreover, it has been shown by J. H. Smith [8] that for any conditionally convergent real series and any real number, there is a rearrangement of a prescribed "cycle type" which converges to that number.

Yet, there obviously are rearrangements which *do* preserve the convergence and sum of all infinite series, whether they converge absolutely or merely conditionally. For example, if the series, $u_1+u_2+u_3+\cdots$, converges, then the rearrangement, $u_2+u_3+u_1+u_5+u_6+u_4+\cdots$, is easily seen to converge to the same sum. It would seem reasonable to try to characterize those rearrangements of series which preserve sums of convergent series. This paper surveys the several approaches to the problem to date and gives another characterization of such rearrangements. For the convenience of would-be series rearrangers, five somewhat simpler sufficient but not necessary conditions for "sum-preserving" rearrangements are also developed. The reader is invited to add to this list.

Rearrangements of series can be described in terms of permutations of the positive integers. Let N denote the set of all positive integers. A *permutation* p of N, of course, is a one-to-one mapping of N onto itself. Let p_j be the image of j under the permutation p. The series Σu_{p_j} is

Paul Schaefer received his Ph.D. from the University of Pittsburgh under George Laush in 1963. He has held faculty positions at the Rochester Institute of Technology and SUNY at Albany, and participated in a faculty exchange at California State University at Los Angeles. Since 1967 he has been Professor of Mathematics at SUNY College at Geneseo. His research interests and publications are in the area of series and summability.—*Editors*