TRENDS IN MATHEMATICS

THE DEATH OF PROOF

by John Horgan, *senior writer*

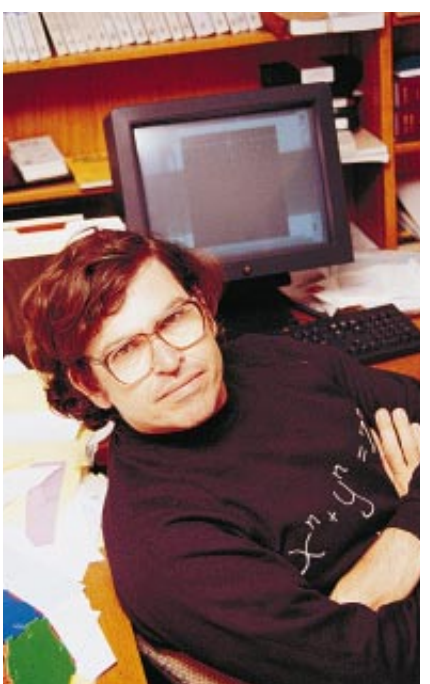
Computers are transforming the way mathematicians discover, prove and communicate ideas, but is there a place for absolute certainty in this brave new world?

Legend has it that when Pythagoras and his followers discovered the theorem that bears his name in the sixth century B.C., they slaughtered an ox and feasted in celebration. And well they might. The relation they found between the sides of a right triangle held true not sometimes or most of the time but always—regardless of whether the triangle was a piece of silk or a plot of land or marks on papyrus. It seemed like magic, a gift from the gods. No wonder so many thinkers, from Plato to Kant, came to believe that mathematics offers the purest truths humans are permitted to know.

That faith seemed reaffirmed this past June when Andrew J. Wiles of Princeton University revealed during a meeting at the University of Cambridge that he had solved Fermat's last theorem. This problem, one of the most famous in mathematics, was posed more than 350 years ago, and its roots extend back to Pythagoras himself. Since no oxen were available, Wiles's listeners showed their appreciation by clapping their hands.

But was the proof of Fermat's last theorem the last gasp of a dying culture? Mathematics, that most tradition-bound of intellectual enterprises, is undergoing profound changes. For millennia, mathematicians have measured progress in terms of what they can demonstrate through proofs—that is, a series of logical steps leading from a set of axioms to an irrefutable conclusion. Now the doubts riddling modern human thought have finally infected mathematics. Mathematicians may at last be forced to accept what many scientists and philosophers already have admitted: their assertions are, at best, only provisionally true, true until proved false.

This uncertainty stems, in part, from the growing complexity of mathematics. Proofs are often so long and complicated that they are difficult to evaluate. Wiles's demonstration runs to 200 pages—and experts estimate it could be five times longer if he spelled out all its elements. One observer as-



"VIDEO PROOF" dramatizes a theorem, proved by William P. Thurston of the Mathematical Sciences Research Institute (left)*, that establishes a profound connection between topology and geometry. The theorem shows how the space surrounding a complex knot (represented by the lattice in this scene) yields a "hyperbolic" geometry, in which parallel lines diverge and the sides of pentagons form right angles. The computer-generated video, called Not Knot, was produced at the Geometry Center in Minnesota.*

serted that only one tenth of 1 percent of the mathematics community was qualified to evaluate the proof. Wiles's claim was accepted largely on the basis of his reputation and the reputations of those whose work he built on. Mathematicians who had not yet examined the argument in detail nonetheless commented that it "looks beautiful" and "has the ring of truth."

Another catalyst of change is the computer, which is compelling mathematicians to reconsider the very nature of proof and, hence, of truth. In recent years, some proofs have required enormous calculations by computers. No mere human can verify these so-called computer proofs, just other computers. Recently investigators have proposed a computational proof that offers only the probability—not the certainty—of truth, a statement that some mathematicians consider an oxymoron. Still others are generating "video proofs" in the hopes that they will be more persuasive than page on page of formal terminology.

At the same time, some mathematicians are challenging the notion that formal proofs should be the supreme standard of truth. Although no one advocates doing away with proofs altogether, some practitioners think the validity of certain propositions may be better established by comparing them with experiments run on computers or with real-world phenomena. "Within the next 50 years I think the importance of proof in mathematics will diminish," says Keith Devlin of Colby College, who writes a column on computers for *Notices of the American Mathematical Society*. "You will see many more people doing mathematics without necessarily doing proofs."

Powerful institutional forces are promulgating these heresies. For several years, the National Science Foundation has been urging mathematicians to become more involved in computer science and other fields with potential applications. Some leading lights, notably Phillip A. Griffiths, director of the Institute for Advanced Study in Princeton, N.J., and Michael Atiyah, who won a Fields Medal (often called the Nobel Prize of mathematics) in 1966 and now heads Cambridge's Isaac Newton Institute for Mathematical Sciences, have likewise encouraged mathematicians to venture forth from their ivory towers and mingle with the real world. At a time when funds and jobs are scarce, young mathematicians cannot afford to ignore these exhortations.

There are pockets of resistance, of course. Some workers are complaining bitterly about the computerization of their field and the growing emphasis on (oh, dirty word) "applications." One of the most vocal champions of tradition is Steven G. Krantz of Washington University. In speeches and articles, Krantz has urged students to choose mathematics over computer science, which he warns could be a passing fad. Last year, he recalls, a National Science Foundation representative came to his university and announced that the agency could no longer afford to support mathematics that was not "goal-oriented." "We could stand up and say this is wrong," Krantz grumbles, "but mathematicians are spineless slobs, and they don't have a tradition of doing that."

David Mumford of Harvard University, who won a Fields Medal in 1974 for research in pure mathematics and is now studying artificial vision, wrote recently that "despite all the hype, the press, the pressure from funding agencies, et cetera, the pure mathematical community by and large still regards

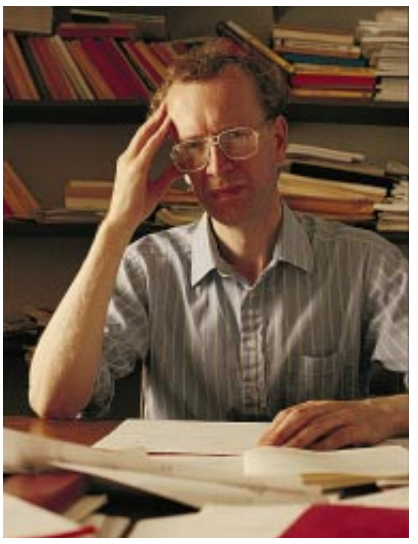
A Splendid Anachronism?

Those who consider experimental mathematics and computer proofs to be abominations rather than innovations have a special reason to delight in the conquest of Fermat's last theorem by Andrew J. Wiles of Princeton University. Wiles's achievement was a triumph of tradition, running against every current in modern mathematics.

Wiles is a staunch believer in mathematics for its own sake. "I certainly wouldn't want to see mathematics just being a servant to applications, because it's not even in the interests of the applications themselves," he says.

The problem he solved, first posed more than 350 years ago by the French polymath Pierre de Fermat, is a glorious example of a purely mathematical puzzle. Fermat claimed to have found a proof of the following proposition: for the equation $X^N + Y^N = Z^N$, there are no integral solutions for any value of N greater than 2. The efforts of mathematicians to find the proof (which Fermat never did disclose) helped to lay the foundation of modern number theory, the study of whole numbers, which has recently become useful in cryptography. Yet Fermat's last theorem itself "is very unlikely to have any applications," Wiles says.

Although funding agencies have been encouraging mathematicians to collaborate, both with each other and with scientists, Wiles worked in virtual solitude for seven years. He shared his ideas with only a few colleagues toward the end of his quest.



Wiles's proof has essentially the same classical, deductive form that Euclid's geometric theorems did. It does not involve any computation, and it claims to be absolutely—not probably—true. Nor did Wiles employ computers to represent ideas graphically, to perform calculations or even to compose his paper; a secretary typed his hand-written notes.

He concedes that testing conjectures with computers may be helpful. In the 1970s computer tests suggested that a far-fetched proposal called the Taniyama conjecture might be true. The tests spurred work that laid the foundation for Wiles's own proof.

Nevertheless, Wiles doubts he will take the trouble to learn how to perform computer investigations. "It's a separate skill," he explains, "and if you're investing that much time on a separate skill, it's quite likely it's taking you away from your real work on the problem."

He rejects the possibility that there may be a finite number of truths accessible to traditional forms of inquiry. "I disagree vehemently with the idea that good theorems are running out," he says. "I think we've barely scratched the surface."

EXPERIMENTAL MATHEMATICIAN Jean E. Taylor of Rutgers University seeks the rules governing minimal surfaces by studying real phenomena, such as soap bubbles, and computer-generated ones, such as idealized crystals (*left*).

computers as invaders, despoilers of the sacred ground." Last year Mumford proposed a course in which instructors would show students how to program a computer to find solutions in advanced calculus. "I was vetoed," he recalled, "and not on the grounds—which I expected—that the students would complain, but because half of my fellow teachers couldn't program!"

That situation is changing fast, if the University of Minnesota's Geometry Center is any indication. Founded two years ago, the Geometry Center occupies the fifth floor of a gleaming, steel and glass polyhedron in Minneapolis. It receives $2 million a year from the National Science Foundation, the Department of Energy and the university. The center's permanent faculty members, most of whom hold positions elsewhere, include some of the most prominent mathematicians in the world.

On a recent day there, several young staff members are editing a video demonstrating how a sphere can be mashed, twisted, yanked and finally turned inside out. In a conference room, three computer scientists from major universities are telling a score of high school teachers how to create computer graphics programs to teach mathematics. Other researchers sit at charcoal-colored NeXT terminals, pondering luridly hued pictures of four-dimensional "hypercubes," whirlpooling fractals and lattices that plunge toward infinity. No paper or pencils are in sight.

At one terminal is David Ben-Zvi, a Harpo Marx–haired junior at Princeton who is spending six months here exploring nonlinear dynamics. He dismisses the fears of some mathematicians that computers will lure them away from the methods that have served them so well for so long. "They're just afraid of change," he says mildly.

The Geometry Center is a hotbed of what is known as experimental mathematics, in which investigators test their ideas by representing them graphically and doing calculations on computers. Last year some of the center's faculty helped to found a journal, *Experimental Mathematics,* that showcases such work. "Experimental methods are not a new thing in mathematics," observes the journal's editor, David B. A. Epstein of the University of Warwick in England, noting that Carl Friedrich Gauss and other giants often performed experimental calculations before constructing formal proofs. "What's new is that it's respectable." Epstein acknowledges that not all his co-workers are so accepting. "One of my colleagues said, 'Your journal should be called the *Journal of Unproved Theorems.*' "

Bubbles and Tortellini

A mathematician who epitomizes the new style of mathematics is Jean E. Taylor of Rutgers University. "The idea that you don't use computers is going to be increasingly foreign to the next generation," she says. For two decades, Taylor has investigated minimal surfaces, which represent the smallest possible area or volume bounded by a curve or surface. Perhaps the most elegant and simple minimal surfaces found in nature are soap bubbles and films. Taylor has always had an experimental bent. Early in her career she tested her handwritten models of minimal surfaces by dunking loops of wire into a sink of soapy water.

Now she is more likely to model bubbles with a sophisticated computer graphics program. She has also graduated from soap bubbles to crystals, which conform to somewhat more complicated rules about minimal surfaces. Together with Frederick J. Almgren of Princeton and Robert F. Almgren of the University of Chicago (her husband and stepson, respectively) and Andrew R. Roosen of the National Institute of Standards and Technology, Taylor is trying to mimic the growth of snowflakes and other crystals on a computer. Increasingly, she is collaborating with materials scientists and physicists, swapping mathematical ideas and programming techniques in exchange for clues about how real crystals grow.

Another mathematician who has prowled cyberspace in search of novel minimal surfaces is David A. Hoffman of the University of Massachusetts at Amherst. Among his favorite quarry are catenoids and helicoids, which resemble the pasta known as tortellini and were first discovered in the 18th century. "We gain a tremendous amount of intuition by looking at images of these surfaces on computers," he says.

In 1992 Hoffman, Fusheng Wei of Amherst and Hermann Karcher of the University of Bonn speculated on the existence of a new class of helicoids, ones with handles. They succeeded in representing these helicoids—the first discovered since the 18th century—on a computer and went on to produce a formal proof of their existence. "Had we not

been able to see a picture that roughly corresponded to what we believed, we would never have been able to do it," Hoffman says.

The area of experimental mathematics that has received the lion's share of attention over the past decade is known as nonlinear dynamics or, more popularly, chaos. In general, nonlinear systems are governed by a set of simple rules that, through feedback and related effects, give rise to complicated phenomena. Nonlinear systems were investigated in the precomputer era, but computers allow mathematicians to explore these systems and watch them evolve in ways that Henri Poincaré and other pioneers of this branch of mathematics could not.

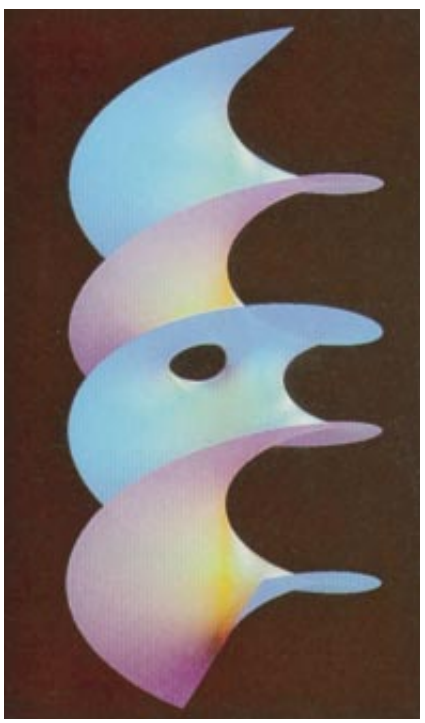
Cellular automata, which divide a computer screen into a set of cells (equivalent to pixels), provide a particularly dramatic illustration of the principles of nonlinearity. In general, the color, or "state," of each cell is determined by the state of its neighbors. A change in the state of a single cell triggers a cascade of changes throughout the system.

One of the most celebrated of cellular automata was invented by John H. Conway of Princeton in the early 1970s. Conway has proved that his automaton, which he calls "Life," is "undecidable": one cannot determine whether its patterns are endlessly variegated or eventually repeat themselves. Scientists have seized on cellular automata as tools for studying the origin and evolution of life. The computer scientist and physicist Edward Fredkin of Boston University has even argued that the entire universe is a cellular automaton.

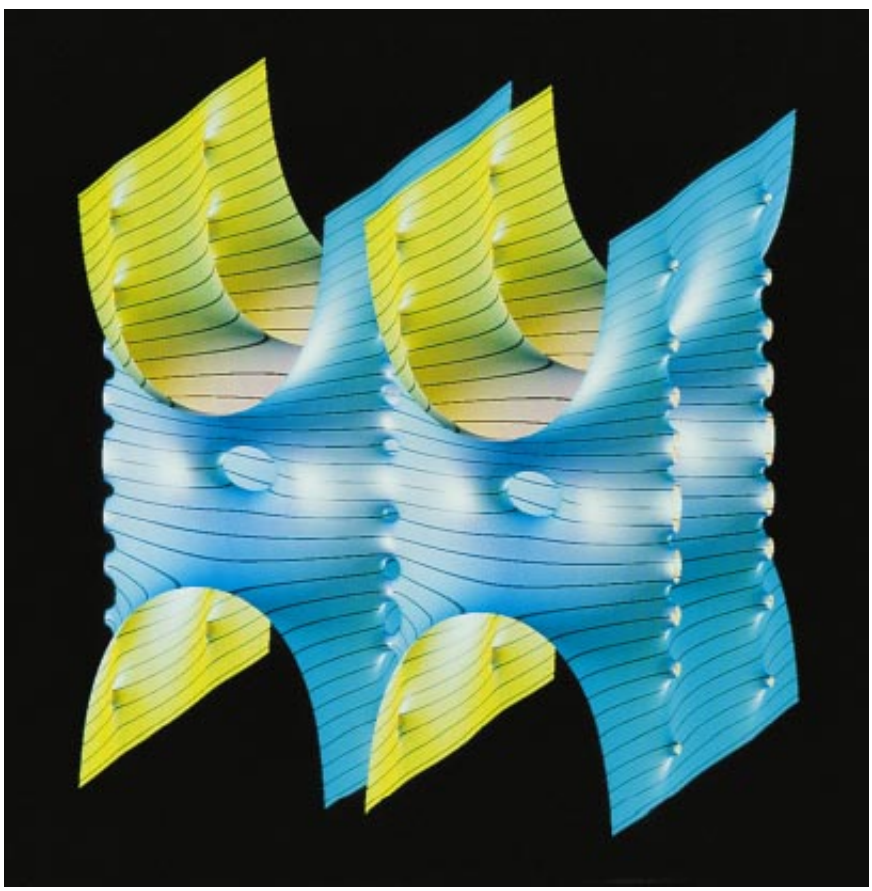
More famous still is the Mandelbrot set, whose image has become an icon for the entire field of chaos since it was popularized in the early 1980s by Benoit B. Mandelbrot of the IBM Thomas J. Watson Research Center. The set stems from a simple equation containing a complex term (based on the square root of a negative number). The equation spits out solutions, which are then iterated, or fed back, into the equation.

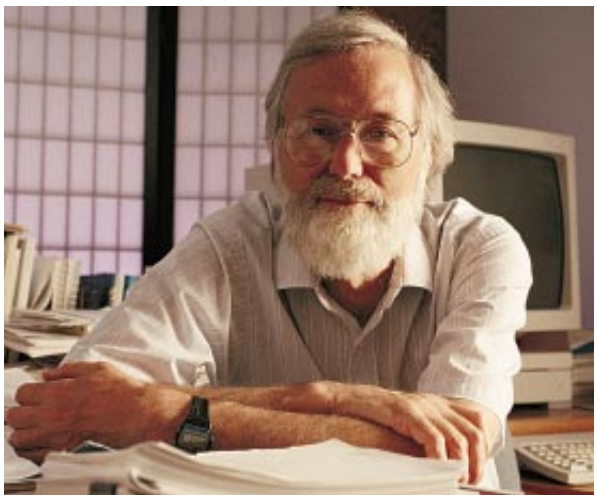
The mathematics underlying the set had been invented more than 70 years ago by two Frenchmen, Gaston Julia and Pierre Fatou, but computers laid bare their baroque beauty for all to see. When plotted on a computer, the Mandelbrot set coalesces into an image that has been likened to a tumorous heart, a badly burned chicken and a warty snowman. The image is a fractal: its fuzzy borders are infinitely long, and it displays patterns that recur at different scales.

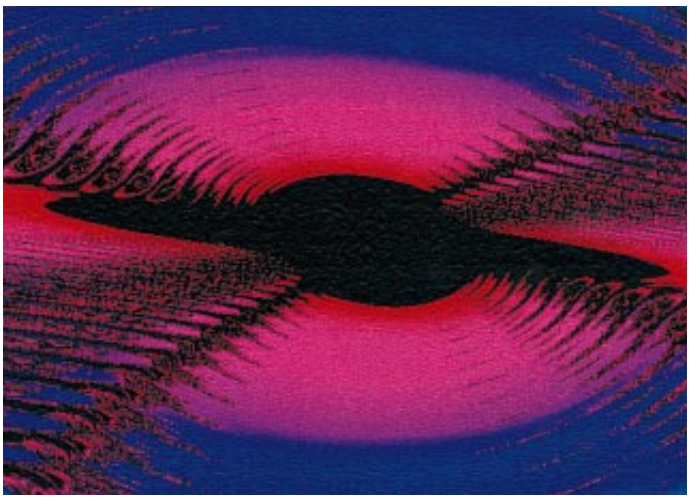
Researchers are now studying sets that are similar to the Mandelbrot set but inhabit four dimensions. "The kinds of complications you get here are the kinds you get in many different sciences," says John Milnor of the State University of New York at Stony Brook. Milnor is trying to fathom the properties of the four-dimensional set by examining two-dimensional slices of it generated by a computer. His preliminary findings led off the inaugural issue of *Experimental Mathematics* last year. Milnor, a 1962 Fields Medalist, says he

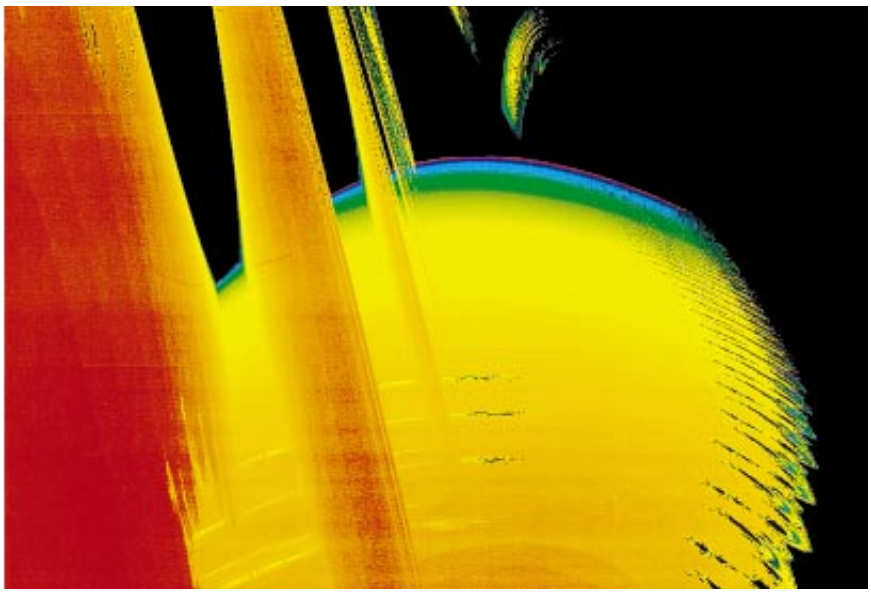


HELICOID WITH A HOLE (*bottom left*) was discovered last year by David A. Hoffman of the University of Massachusetts at Amherst and his colleagues, with the help of computer graphics. Edward C. Thayer, one of Hoffman's graduate students, recently found a structure (*below*) that mimics the pattern of certain polymers.









UNEARTHLY LANDSCAPES emerge when a computer generates "slices" of a four-dimensional map similar to the well-known Mandelbrot set. John Milnor of the State University of New York at Stony Brook studies similar two-dimensional images in order to understand the properties of the complex mathematical object.

occasionally performed computer experiments in the days of punch cards, but "it was a miserable process. It has become much easier."

The popularity of graphics-oriented mathematics has provoked a backlash. Krantz of Washington University charged four years ago in the *Mathematical Intelligencer* that "in some circles, it is easier to obtain funding to buy hardware to generate pictures of fractals than to obtain funding to study algebraic geometry."

A broader warning about "speculative" mathematics was voiced this past July in the *Bulletin of the American Mathematical Society* by Arthur Jaffe of Harvard and Frank S. Quinn of the Virginia Polytechnic Institute. They suggested that computer experiments and correspondence with natural phenomena are no substitute for proofs in establishing truth. "Groups and individuals within the mathematics community have from time to time tried being less compulsive about details of arguments," Jaffe and Quinn wrote. "The results have been mixed, and they have occasionally been disastrous."

Most mathematicians exploiting computer graphics and other experimental techniques agree that seeing should not be believing and that proofs are still needed to verify the conjectures they arrive at through computation. "I think mathematicians were contemplating their navels for too long, but that doesn't mean I think proofs are irrelevant," Taylor says. Hoffman offers an even stronger defense of traditional proofs. "Proofs are the only laboratory instrument mathematicians have," he remarks, "and they are in danger of being thrown out." Although computer graphics are "unbelievably wonderful," he adds, "in the 1960s drugs were unbelievably wonderful, and some people didn't survive."

Indeed, veteran computer enthusiasts know better than most that computational experiments—whether involving graphics or numerical calculations—can be deceiving. One cautionary tale involves the Riemann hypothesis, a famous prediction about the patterns displayed by prime numbers as they march toward infinity. First posed more than 100 years ago by Bernhard Riemann, the hypothesis is considered to be one of the most important unsolved problems in mathematics.

A contemporary of Riemann's, Franz Mertens, proposed a related conjecture involving positive whole numbers; if true, the conjecture would have provided strong evidence that the Riemann hypothesis was also true. By the early 1980s computers had shown that Mertens's proposal did indeed hold for at least the first 10 billion integers. In 1984, however, more extensive computations revealed that eventually—at numbers as high as $10^{(10^{70})}$—the pattern predicted by Mertens vanishes.

One potential drawback of computers is that all their calculations are based on the manipulation of discrete, whole numbers—in fact, ones and zeros. Computers can only approximate real numbers, such as *pi* or the square root of two. Someone knowledgeable about the rounding-off functions of a simple pocket calculator can easily induce it to generate incorrect answers to calculations. More sophisticated programs can make more complicated and elusive errors. In 1991 David R. Stoutemyer, a software specialist at the University of Hawaii, presented 18 experiments in algebra that gave wrong an-

swers when performed with standard mathematics software.

Stephen Smale of the University of California at Berkeley, a 1966 Fields Medalist, has sought to place mathematical computation on a more secure foundation—or at least to point out the size and location of the cracks running through the foundation. Together with Lenore Blum of the Mathematical Sciences Research Institute at Berkeley and Michael Shub of IBM, he has created a theoretical model of a computer that can process real numbers rather than just integers.

Blum and Smale recently concluded that the Mandelbrot set is, in a technical sense, uncomputable. That is, one cannot determine with certainty whether any given point on the complex plane resides within or outside the set's hirsute border. These results suggest that "you have to be careful" in extrapolating from the results of computer experiments, Smale says.

These concerns are dismissed by Stephen Wolfram, a mathematical physicist at the University of Illinois. Wolfram is the creator of Mathematica, which has become the leading mathematics software since first being marketed five years ago. He acknowledges that "there are indeed pitfalls in experimental mathematics. As in all other kinds of experiments, you can do them wrong." But he emphasizes that computational experiments, intelligently performed and analyzed, can yield more results than the old-fashioned conjecture-proof method. "In every other field of science there are a lot more experimentalists than theorists," Wolfram says. "I suspect that will increasingly be the case with mathematics."

"The obsession with proof," Wolfram declares, has kept mathematicians from discovering the vast new realms of phenomena accessible to computers. Even the most intrepid mathematical experimentalists are for the most part "not going far enough," he says. "They're taking existing questions in mathematics and investigating those. They are adding a few little curlicues to the top of a gigantic structure."

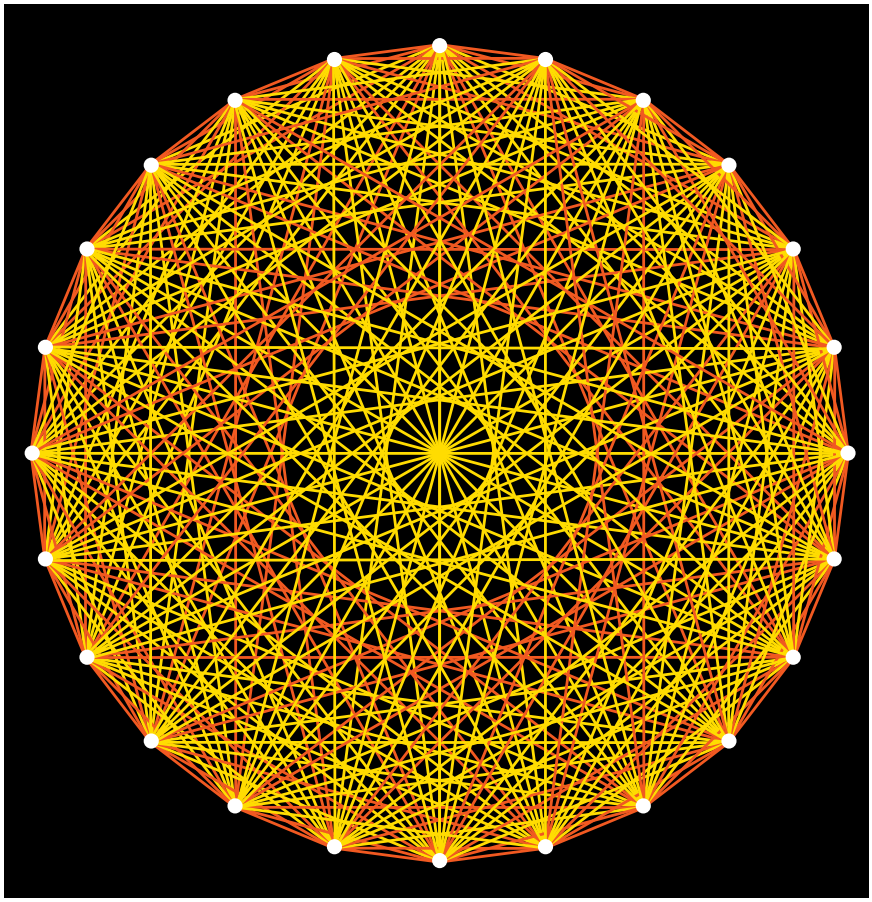
Mathematicians may take this view with a grain of salt. Although he shares Wolfram's fascination with cellular automata, Conway contends that Wolfram's career—as well as his contempt for proofs—shows he is not a real mathematician. "Pure mathematicians usually don't found companies and deal with the world in an aggressive way," Life's creator says. "We sit in our ivory towers and think about things."

Purists may have a harder time ignoring William P. Thurston, who is also an enthusiastic booster of experimental mathematics and of computers in mathematics. Thurston, who heads the Mathematical Sciences Research Institute at Berkeley and is a co-director of the Geometry Center (with Albert Marden of the University of Minnesota), has impeccable credentials. In the mid-1970s he pointed out a deep potential connection between two separate branches of mathematics—topology and geometry. Thurston won a Fields Medal for this work in 1982.

Thurston emphasizes that he believes mathematical truths are discovered and not invented. But on the subject of proofs, he sounds less like a disciple of Plato than of Thomas S. Kuhn, the philosopher who argued in his 1962 book, *The Structure of Scientific Revolutions,* that scientific theories are accepted for social reasons rather than because they are in any objective sense "true." "That mathematics reduces in principle to formal proofs is a shaky idea" peculiar to this century, Thurston asserts. "In practice, mathematicians prove theorems in a social context," he says. "It is a socially conditioned body of knowledge and techniques."

The logician Kurt Gödel demonstrated more than 60 years ago through his incompleteness theorem that "it is impossible to codify mathematics," Thurston notes. Any set of axioms yields statements that are self-evidently true but cannot be demonstrated with those axioms. Bertrand Russell pointed out even earlier that set theory, which is the basis of much of mathematics, is rife with logical contradictions related to the problem of self-reference. (The self-contradicting statement "This sentence is false" illustrates the problem.) "Set theory is based on polite lies, things we agree on even though we know they're not true," Thurston says. "In some ways, the foundation of mathematics has an air of unreality."

Thurston thinks highly formal proofs are more likely to be flawed than those appealing to a more intuitive level of understanding. He is particularly enamored of the ability of computer graphics



PARTY PROBLEM was solved after a vast computation by Stanislaw P. Radziszowski and Brendan D. McKay. They calculated that at least 25 people are required to ensure either that four people are all mutual acquaintances or that five are mutual strangers. This diagram, in which red lines connect friends and yellow lines link strangers, shows that a party of 24 violates the dictum.

Silicon Mathematicians

The continuing penetration of computers into mathematics has revived an old debate: Can mathematics be entirely automated? Will the great mathematicians of the next century be made of silicon?

In fact, computer scientists have been working for decades on programs that generate mathematical conjectures and proofs. In the late 1950s the artificial-intelligence guru Marvin Minsky showed how a computer could "rediscover" some of Euclid's basic theorems in geometry. In the 1970's Douglas Lenat, a former student of Minsky's, presented a program that devised even more advanced geometry theorems. Skeptics contended that the results were, in effect, embedded in the original program.

A decade ago the computer scientist and entrepreneur Edward Fredkin sought to revive the sagging interest in machine mathematics by creating what came to be known as the Leibniz Prize. The prize, administered by Carnegie Mellon University, offers $100,000 for the first computer program to devise a theorem that has a "profound effect" on mathematics.

Some practitioners of what is known as automated reasoning think they may be ready to claim the prize. One is Larry Wos of Argonne National Laboratory, editor of the *Journal of Automated Reasoning.* He claims to have developed a program that has solved problems in mathematics and logic "that have stumped people for years." Another is Siemeon Fajtlowicz of the University of Houston, inventor of a program, called Graffiti, that has proposed "thousands" of conjectures in graph theory.

None of these achievements comes close to satisfying the "profound effect" criterion, according to David Mumford of Harvard University, a judge for the prize. "Not now, not 100 years from now," Mumford replies when asked to predict when the prize might be claimed.

Some observers think computers will eventually surpass our mathematical abilities. After all, notes Ronald L. Graham of AT&T Bell Laboratories, "we're not very well adapted for thinking about the space-time continuum or the Riemann hypothesis. We're designed for picking berries or avoiding being eaten."

Others side with the mathematical physicist Roger Penrose of the University of Oxford, who in his 1989 book, *The Emperor's New Mind,* asserted that computers can never replace mathematicians. Penrose's argument drew on quantum theory and Gödel's incompleteness theorem, but he may have been most convincing when discussing his personal experience. At its best, he suggested, mathematics is an art, a creative act, that cannot be reduced to logic any more than *King Lear* or Beethoven's Fifth can.

to communicate abstract mathematical concepts to others both within and outside the professional community. Two years ago, at his urging, the Geometry Center produced a computer-generated "video proof," called Not Knot, that dramatizes a ground-breaking conjecture he proved a decade ago [*see illustration on pages 92 and 93*]. Thurston mentions proudly that the rock band the Grateful Dead has shown the Not Knot video at its concerts.

Whether Deadheads grok the substance of the video—which concerns how mathematical objects called three-manifolds behave in a non-Euclidean "hyperbolic" space—is another matter. Thurston concedes that the video is difficult for nonmathematicians, and even some professionals, to fathom, but he is undaunted. The Geometry Center is now producing a video of yet another of his theorems, which demonstrates how a sphere can be turned inside out [*see cover illustration*]. Last fall, moreover, Thurston organized a workshop at which participants discussed how virtual reality and other advanced technologies could be adapted for mathematical visualization.

Paradoxically, computers have catalyzed a countertrend in which truth is obtained at the expense of comprehensibility. In 1976 Kenneth Appel and Wolfgang Haken of the University of Illinois claimed they had proved the four-color conjecture, which stated that four hues are sufficient to construct even an infinitely broad map so that no identically colored countries share a border. In some respects, the proof of Appel and Haken was conventional—that is, it consisted of a series of logical, traceable steps proceeding to a conclusion. The conclusion was that the conjecture could be reduced to a prediction about the behavior of some 2,000 different maps.

Since checking this prediction by hand would be prohibitively time-consuming, Appel and Haken programmed a computer to do the job for them. Some 1,000 hours of computing time later, the machine concluded that the 2,000 maps behave as expected: the four-color conjecture was true.

The Party Problem

Other computer-assisted proofs have followed. Just this year, a proof of the so-called party problem was announced by Stanislaw P. Radziszowski of the Rochester Institute of Technology and Brendan D. McKay of the Australian National University in Canberra. The problem, which derives from work in set theory by the British mathematician Frank P. Ramsey in the 1920s, can be phrased as a question about relationships between people at a party. What is the minimum number of guests that must be invited to guarantee that at least X people are all mutual acquaintances or at least Y are mutual strangers? This number is known as a Ramsey number.

Previous proofs had established that 18 guests are required to ensure that there are either four mutual acquaintances or four strangers. In their proof, Radziszowski and McKay showed that the Ramsey number for four friends or five strangers is 25. Socialites might think twice about trying to calculate the Ramsey number for greater X's and Y's. Radziszowski and McKay estimate that their proof consumed the equivalent of 11 years of computation by a standard desktop machine. That may be a record, Radziszowski says, for a problem in pure mathematics.

The value of this work has been debated in an unlikely forum—the newspaper column of advice-dispenser Ann Landers. In June a correspondent complained to Landers that resources spent on the party problem should have been used to help "starving children in wartorn countries around the world." Some mathematicians raise another objection to computer-assisted proofs. "I don't believe in a proof done by a computer, says Pierre Deligne of the Institute for Advanced Study, an algebraic geometer and 1978 Fields Medalist. "In a way, I am very egocentric. I believe in a proof if I understand it, if it's clear." While recognizing that humans can make mistakes, he adds: "A computer will also

make mistakes, but they are much more difficult to find."

Others take a more functional point of view, arguing that establishing truth is more important than giving mathematicians an aesthetic glow, particularly if a result is ever to find an application. Defenders of this approach, who tend to be computer scientists, point out that conventional proofs are far from immune to error. At the turn of the century, most theorems were short enough to read in one sitting and were produced by a single author. Now proofs often extend to hundreds of pages or more and are so complicated that years may pass before they are confirmed by others.

The current record holder of all conventional proofs was completed in the early 1980s and is called the classification of finite, simple groups. (A group is a set of elements, such as integers, together with an operation, such as addition, that combines two elements to get a third one.) The demonstration consists of some 500 articles totaling nearly 15,000 pages and written by more than 100 workers. It has been said that the only person who grasped the entire proof was its general contractor, Daniel Gorenstein of Rutgers. Gorenstein died last year.

Much shorter proofs can also raise doubts. Three years ago Wu-Yi Hsiang of Berkeley announced he had proved an old conjecture that one can pack the most spheres in a given volume by stacking them like cannonballs. Today some skeptics are convinced the 100-page proof is flawed; others are equally certain it is basically correct.

Indeed, the key to greater reliability, according to some computer scientists, is not less computerization but more. Robert S. Boyer of the University of Texas at Austin has led an effort to squeeze the entire sprawling corpus of modern mathematics into a single data base whose consistency can be verified through automated "proof checkers."

The manifesto of the so-called QED Project states that such a data base will enable users to "scan the entirety of mathematical knowledge for relevant results and, using tools of the QED system, build upon such results with reliability and confidence but without the need for minute comprehension of the details or even the ultimate foundations." The QED system, the manifesto proclaims rather grandly, can even "provide some antidote to the degenerative effects of cultural relativism and nihilism" and, presumably, protect mathematics from the all-too-human willingness to succumb to fashion.

The debate over computer proofs has intensified recently with the advent of a technique that offers not certainty but only a statistical probability of truth. Such proofs exploit methods similar to those underlying error-correction codes, which ensure that transmitted messages are not lost to noise and other effects by making them highly redundant. The proof must first be spelled out precisely in a rigorous form of mathematical logic. The logic then undergoes a further transformation called arithmetization, in which "and," "or" and other functions are translated into arithmetic operations, such as addition and multiplication.

Like a message transformed by an error-correction code, the "answer" of a probabilistic demonstration is distributed throughout its length—as are any errors. One checks the proof by querying it at different points and determining whether the answers are consistent; as the number of checks increases, so does the certainty that the argument is correct. Laszlo Babai of the University of Chicago, who developed the proofs two years ago (along with Lance Fortnow, Carsten Lund and Mario Szegedy of Chicago and Leonid A. Levin of Boston University), calls them "transparent." Manuel Blum of Berkeley, whose work helped to pave the way for Babai's group, suggests the term "holographic."

The Uncertain Future

Whatever they are named, such proofs have practical drawbacks. Szegedy acknowledges that transforming a conventional demonstration into the probabilistic form is difficult, and the result can be a "much bigger and uglier animal." A 1,000-line proof, for example, could easily balloon to $1{,}000^3$ (1,000,000,000) lines. Yet Szegedy contends that if he and his colleagues can simplify the transformation process, probabilistic proofs might become a useful method for verifying mathematical propositions and large computations—such as those leading to the four-color theorem. "The philosophical cost of this efficient method is that we lose the absolute certainty of a Euclidean proof," Babai noted in a recent essay. "But if you do have doubts, will you bet with me?"

Such a bet would be ill advised, Levin believes, since a relatively few checks can make the chance of error vanishingly small: one divided by the number of particles in the universe. Even the most straightforward conventional proofs, Levin points out, are susceptible to doubts of this scale. "At the moment you find an error, your brain may disappear because of the Heisenberg uncertainty principle and be replaced

by a new brain that thinks the proof is correct," he says.

Ronald L. Graham of AT&T Bell Laboratories suggests that the trend away from short, clear, conventional proofs that are beyond reasonable doubt may be inevitable. "The things you can prove may be just tiny islands, exceptions, compared to the vast sea of results that cannot be proved by human thought alone," he explains. Mathematicians seeking to navigate uncharted waters may become increasingly dependent on experiments, probabilistic proofs and other guides. "You may not be able to provide proofs in a classical sense," Graham says.

Of course, mathematics may yield fewer aesthetic satisfactions as investigators become more dependent on computers. "It would be very discouraging," Graham remarks, "if somewhere down the line you could ask a computer if the Riemann hypothesis is correct and it said, 'Yes, it is true, but you won't be able to understand the proof.'"

Traditionalists no doubt shudder at the thought. For now, at least, they can rally behind heros like Wiles, the conqueror of Fermat's last theorem, who eschews computers, applications and other abominations. But there may be fewer Wileses in the future if reports from the front of precollege education are any guide. The Mathematical Sciences Research Institute at Berkeley, which is overseen by Thurston, has been holding an ongoing series of seminars with high school teachers to find new ways to entice students into mathematics. This past January Lenore Blum, the institute's deputy director, organized a seminar devoted to the question "Are Proofs in High School Geometry Obsolete?"

The mathematicians insisted that proofs are crucial to ensure that a result is true. The high school teachers demurred, pointing out that students no longer considered traditional, axiomatic proofs to be as convincing as, say, visual arguments. "The high school teachers overwhelmingly declared that most students now (Nintendo/joystick/MTV generation) do not relate to or see the importance of 'proofs,'" the minutes of the meeting stated. Note the quotation marks around the word "proofs."

FURTHER READING

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