How to Do and Write Math Research

REUBEN HERSH

any experienced researchers could write a better article on the topic. But they didn't write this, so I did. This advice is for the newcomer, the beginner, who may have cried in the night, "How the heck do you do math research, anyhow?"

I deal separately with two topics.
How do you do it?
How do you write it?

How Do You Do Research?

Where do you get the idea? Or, as it's often put, where do you find your problems? There are several places, some worse than others.

Let us assume you already have an "adviser" and a "field" or "area," and your adviser has already told you that this year's hot problem in your area is say "the Uniqueness Problem." Maybe your thesis was a partial solution—uniqueness subject to Condition A. Your first publication!

What next?

1. One popular strategy is, chip away at Condition A. In your next paper, weaken it to Condition A'. And then condition A'' for the paper after that. You'll quite possibly become known as an expert on weakening Condition A.

2. Another idea is to keep your eye on the Uniqueness Problem itself. Half a dozen other hot shots and sharpies like you are already in the race. Keep up with them by e-mail or face to face. See how to strengthen X's new trick by simply refining his metric and then bringing in Y's old trick from last year. Write it up fast, to get it out before Z.

If you have speed and stamina, you may become known as one of the active young generation working on the Uniqueness Problem.

3. There are still other ways to go. You may struggle to remedy the deficiencies in your education, which you only now recognize. You may start to look at journals outside your field. If you behave in this non-standard way, you will occasionally spot an anomaly. A curious parallel between seemingly unrelated results, or an unnoticed, unexploited regularity.

Figure out what's going on. You'll end up with a publication, even several publications. You may then notice a crowd of eager youngsters peeking over your shoulder, looking for generalizations and specializations and extensions of your idea.

4. Generalization. A much-traveled high road to publication. Professor Q has a result on L^p-spaces. Generalize it to abstract Banach space. Then again, to Fréchet spaces. Then perhaps still again, even to Hausdorff spaces. Three solid papers in just one month. If you had been thoughtless, you could have done Hausdorff in the first place. But that would have been only one paper. I warn you—this is not the way to go! You'll acquire a life-long reputation as a trifler and nose-picker.

Before you generalize, ask two questions. Does the generalization include at least one interesting special case that wasn't already covered? Does proving the generalization require a new idea, not standard in the generalized context? If the answer to both is No, keep it for a dissertation problem for your first marginal, weak student.

5. Specialization is underrated as a research tactic. If Professor Y has a nice theorem on Banach space, does bringing it down to L^p yield some surprising concrete formulas, or some unexpected connection with probability or p.d.e.?

Of these five roads, linking hitherto unrelated fields or methods or results is the one most apt to be a winner. It does require you to know a little about two different areas.

If you like to hide in your office till your proofs are all complete and then astonish your colleagues, you're at a serious disadvantage. Talking to others can be helpful. It may be hard to find anybody who'll listen. One way is to find somebody else who's also looking for somebody to listen. She listens about your sub-von Neumannian hyper-loops, you listen about her semi-Markovian submartingales.

Writing It

Your first big decision is: are you writing to be understood, or to not be understood?

In a letter to Florimond De Beaune, René Descartes wrote, on February 20, 1639, ". . . in the case of the tangents, I have only given a simple example of analysis, taken indeed from a rather difficult aspect and I have left out many of the things which could have been added so as to make the practice of the analysis more easy. I can assure you, nevertheless, that I have omitted all that quite deliberately, except in the case of the asymptote, which I forgot. But I felt sure that certain people, who boast that they know everything, would not miss the chance of saying that they knew already what I had written, if I had made myself easily intelligible to them. I should not then have had the pleasure, which I have since enjoyed, of noting the irrelevance of their objections."

Some mathematicians write so as not to be understood, without consciously recognizing that as their goal. Sleepwalking has its advantages. But since you're reading this paper, I presume you want to know what you're doing.

Some tips on how not to be understood, gleaned from the recent literature.

1) Don't explain why you're doing what you're doing, beyond a cryptic remark like "The Uniqueness Problem is related to the construction and classification problems for sub-von Neumannian hyper-loops."

2) Don't explain in any meaningful way what has already been done. Do give references—at least 30, mostly in French.

3) Avoid natural language. Anything that can be said in English can be said in symbols, if you make an honest effort.

4) Don't repeat anything already stated in any of your references. Not many readers will track them down in the library, especially if your references aren't in the library.

5) Use many different type faces. Gothic is good. But double and triple subscripts may be rejected by the printer.

6) Write "it's easy to see" and "a short calculation shows" and "it follows immediately" and "by a well-known argument of Nicomachus" as often as you dare.

So much for that.

What if you want to be understood?

Writing to be understood is more trouble than writing not to be understood. Over your lifetime, it will cut down your "productivity." There's a popular belief that understandable writing is bad for your reputation. "If I can understand it, it must be trivial." Don't be intimidated by this belief. Defy it. Mathematicians are grateful to mathematicians who write understandably. They are understandably more likely to read understandable papers.

Writing understandably means, of course, *not* following the practices recommended above for being not understood.

A good way to write an understandable paper is to have one central idea. State it at the beginning. Carry it out in successive stages. Point out as you go that you're carrying out the central idea you stated at the beginning.

Your work may not have such a unified, coherent character. Maybe you did several interrelated things, all relevant to the Uniqueness Problem. Then make a clear separation of your paper into parts, each of which is unified. In the introduction, explain why you have organized the paper as you have. Where different sections of the paper are connected, discuss the connection in *both* places. Taking up a few lines of print to save your reader a few minutes of trouble is legitimate.

AUTHOR

REUBEN HERSH
1000 Camino Rancheros
Santa Fe, NM 87501
USA
e-mail:rhersh@math.unm.edu

Reuben Hersh has published, in addition to his articles of mathematics, many articles *about* mathematics, in *Scientific American*, *The Mathematical Intelligencer*, and widely read books. His new book, *What is Mathematics, Really?*, is scheduled for publication in 1997 (Oxford University Press). He is retired and lives in Santa Fe, NM. He is divorced and has two grandchildren. Here he is shown with his friend and student, Laura John.

If you want to omit an argument or a calculation, give a short outline of what you're omitting, so that an interested reader really has a chance to fill it in.

Use the highest standards of composition of which you are capable. Never a pronoun whose reference is fuzzy in the slightest. Short words rather than long. Concrete nouns and verbs rather than abstract. Active voice rather than passive. Short, grammatical sentences. Paragraphing according to the flow of ideas. Be consistent in using the past tense or the present, in having as a subject "you" or "we" or "one."

If you use a foreign word, do tell us what it means.

When you're finished, put your paper away for a week. Reread it. *In each sentence*, look for unnecessary words. Cross them out.

In each paragraph, look for unnecessary sentences. Cross them out.

Look especially for unexplained terms and concepts. Explain them.

Put your paper away for another week.

Reread it. Correct it again.

Continue putting it away for a week, rereading it and correcting it, until it is perfect.

You've done your duty. Send it in.

Expect complaints and requests for more changes.

REFERENCE

Descartes, R. *Oeuvres*, ed. C. Adam and P. Tannery, 12 vols., Paris, L. Cert, 1897–1910, ii 511, 18–20; Corr. III, 185

Mixed Motives

B. Sury

The train jiggled as it ran, and gave periodic shakes like a man caught without his sweater on a cold night. In the second-class compartment, Monsieur Pi Rho was covertly studying the passenger opposite him. The young man had bloodshot eyes; evidently he hadn't slept properly for many days. He was bespectacled and clad in a Khadi Kurta and jeans which hadn't seen water for a considerable period of time; obviously a man of spartan tastes in the matter of attire. His eyes gleamed with a ferocity that spoke of some deep and sinister purpose. Every once in a while, he would jump out of his seat in his restlessness and walk along the aisle, cross the vestibule, and return after a five-minute stroll.

Monsieur Pi Rho (though having retired from his profession two years back) could place him unerringly. Yes, his instincts told him that the young man was up to something, which, whatever it was, would not be too long in coming.

The dinner was long over, most of the lights had been put off, people had given up the pretense of reading, and indeed most of them were snoring gently. The young man took out a packet of cigarettes and moved toward the door. Monsieur Pi Rho got up and, peeping surreptitiously out of the compartment, saw the young man begin his smoke. Tiptoeing back, he pulled from under the seat the dirty-looking brown bag which seemed to be the young man's only piece of baggage. He quickly unzipped it and looked inside. In the dim light, he could see sheets of paper.

Monsieur Pi Rho pulled out one and peered at some handwritten instructions. He began reading.

"We are provided with a scheme and a map which is proper to a point," read Monsieur Pi Rho. With widening eyes, he skimmed through the page to see if any person or place was mentioned. Beyond cryptic words like "the group can act freely but discreetly," (the last word had been misspelt), "go to a cover to kill the classes," and "blow-up if necessary," nothing specific was mentioned. Evidently, some group was planning an ambush, but where and on whom? Who were they? (These people expressed themselves in a strange Pickwickian language—surely in the interest of security.)

Monsieur Pi Rho didn't have too much time before the young man's return. He quickly turned a couple of pages and saw the heading "Motives." Here the language was even more exasperatingly vague. There was again mention of a group whose representatives were deemed to be traceless. Also mentioned was a corpse (also misspelt) which was totally disconnected (!), on which some functions still existed but were rapidly decreasing. Whose could it be? And where? Presumably in some local field. Somehow the job of this corpse was threefold:

(i) split some (presumably rival) group;

(ii) decompose certain representatives of the group; and

(iii) infiltrate by powers of ideals.

These people even talked of ideals! Monsieur Pi Rho pondered for a moment on this mysterious group's ways. He turned to the last page and THEN HE KNEW! There it was clearly written: "BULLET IN THE AMERICAN MATHEMATICAL SOCIETY!"

Hastily thrusting the manuscript back in its bag, Monsieur Pi Rho returned innocuously to his seat. He must act quickly. What should he do? Would confronting the youth solve the problem? Involving, as it did, such a radical group, solvability depended on their Killing form. He came to a quick decision. No! He would wait until the train reached Hampur and wire for help. The critical point was to use Morse. THAT would certainly put them in their cells.

School of Mathematics
Tata Institute of Fundamental Research
Bombay, 400 005
India