THE GRADED GROTHENDIECK GROUP AS A CLASSIFICATION TOOL FOR ALGEBRAS, CURRENT STATUS

It is conjectured [3] that the graded Grothendieck group is a complete invariant for Leavitt path algebras. This was established for the class of Leavitt path algebras associated to polycephaly graphs [3]. One can write the analytic version of this conjecture and it is hoped that it would be easier to check the analytic version for graph C^*-algebras. In following we give a brief summary of the current status of the program. All Leavitt path algebras are defined over an arbitrary field.

Conjecture 1. *Let E and F be finite graphs. Then there is an order preserving $\mathbb{Z}[x,x^{-1}]$-module isomorphism*

$$\phi : K_0^{\rm gr}(L(E)) \longrightarrow K_0^{\rm gr}(L(F))$$

with $\phi([L(E)] = [L(F)]$ if and only if $L(E) \cong_{\rm gr} L(F)$.

Here the ordered $\mathbb{Z}[x,x^{-1}]$-module isomorphism $K_0^{\rm gr}(L(E)) \cong K_0^{\rm gr}(L(F))$ should give that these algebras are graded Morita equivalent (see the diagram below).

Denote by γ_E the gauge circle actions on $C^*(E)$ and $K_0^{\mathbb{T}}(C^*(E))$ the equivaraint K-theory of $C^*(E)$. There are canonical ordered isomorphisms of $\mathbb{Z}[x,x^{-1}]$-modules (see [5])

$$K_0^{\rm gr}(L(E)) \cong K_0(L(E\times\mathbb{Z})) \cong K_0(C^*(E\times\mathbb{Z})) \cong K_0^{\mathbb{T}}(C^*(E)). \qquad (1)$$

Thus one can pose the analytic version of Conjecture 1 as follows.

Conjecture 2. *Let E and F be finite graphs. Then there is an order preserving $\mathbb{Z}[x,x^{-1}]$-module isomorphism*

$$\phi : K_0^{\mathbb{T}}(C^*(E)) \longrightarrow K_0^{\mathbb{T}}(C^*(F)),$$

with $\phi([C^*(E)]) = [C^*(F)]$ if and only if $C^*(E) \cong C^*(F)$ which respect the gauge action.

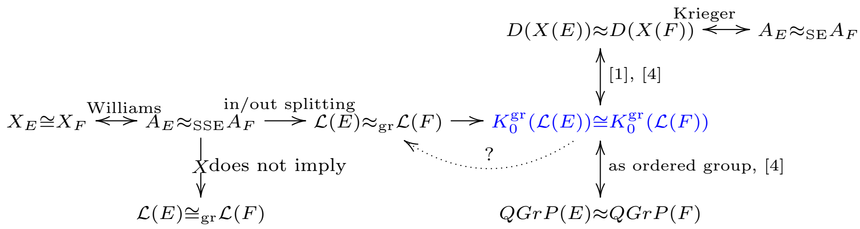
In fact in Conjecture 1 if the graded Grothendieck groups are isomorphic, one should have the isomorphism between the Leavitt path algebras are indeed $*$-isomorphism. If this is the case, then Conjecture 1 implies Conjecture 2. For, if $K_0^{\mathbb{T}}(C^*(E)) \cong K_0^{\mathbb{T}}(C^*(F))$, then by (1), $K_0^{\rm gr}(L(E)) \cong K_0^{\rm gr}(L(F))$, so $L(E) \cong_{\rm gr} L(F)$ as $*$-isomorphism. This implies that $C^*(E) \cong C^*(F)$ which respects the gauge action. ([1, Theorem 4.4]).

We can summarise the current status of the conjectures in the following diagram:

Notations:

- E and F are finite graphs, A_E and A_F the adjacency matrices and X_E and X_F are associated shift of finite types.
- SSE stands for *strongly shift equivalent*; SE stands for *shift equivalent*.
- $D(X(E)) = (\Delta_A, \Delta_A^+, \delta_A)$ is the Krieger's dimension group associated to the matrix A.

• $\cong_{\text{gr}}$ denotes the graded isomorphism; $\approx_{\text{gr}}$ denoted the graded Morita equivalent.



In the diagram above, it should be possible to replace K_0^{gr} by $K_0^{\mathbb{T}}$ and $L(E)$ and $L(F)$ by $C^*(E)$ $C^*(F)$ and all statements would still hold (see Conjecture 2).

References

[1] G. Abrams, M. Tomforde, *Isomorphism and Morita equivalence of graph algebras*, Trans. Amer. Math. Soc. **363** (2011), no. 7, 3733–3767.

[2] P. Ara, E. Pardo, *Towards a K-theoretic characterization of graded isomorphisms between Leavitt path algebras,* J. K-Theory **14** (2014), no. 2, 203–245.

[3] R. Hazrat, *The graded Grothendieck groups and the classification of Leavitt path algebras,* Math Annalen, **355** (2013), no. 1, 273–325.

[4] R. Hazrat, *The dynamics of Leavitt path algebras*, Journal of Algebra, **384** (2013) 242–266.

[5] R. Hazrat, *A note on the isomorphism conjectures for Leavitt path algebras,* Journal of Algebra **375** (2013) 33–40.

[6] W. Krieger, *On dimension functions and topological Markov chains*, Invent. Math. **56** (1980), no. 3, 239–250.